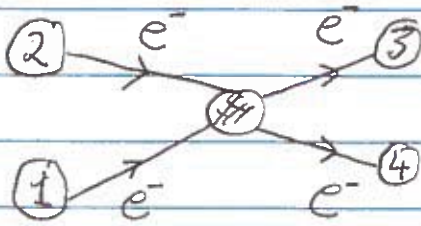


1.



Electrons created at ① & ② $\Rightarrow \bar{\psi}(1) \bar{\psi}(2)$

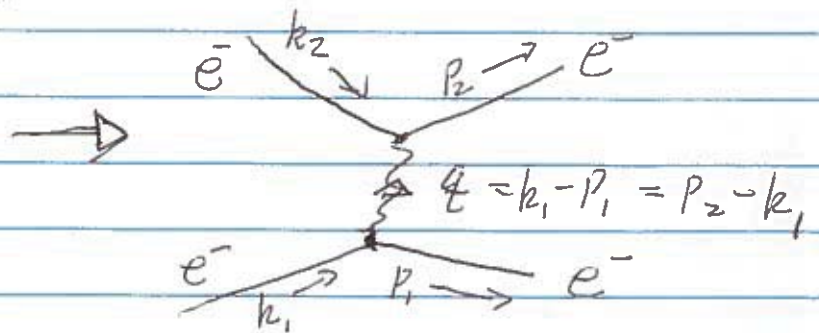
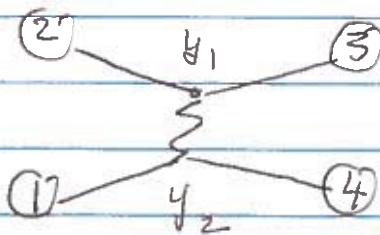
Electrons annihilated at ③ & ④ $\Rightarrow \psi(3) \psi(4)$

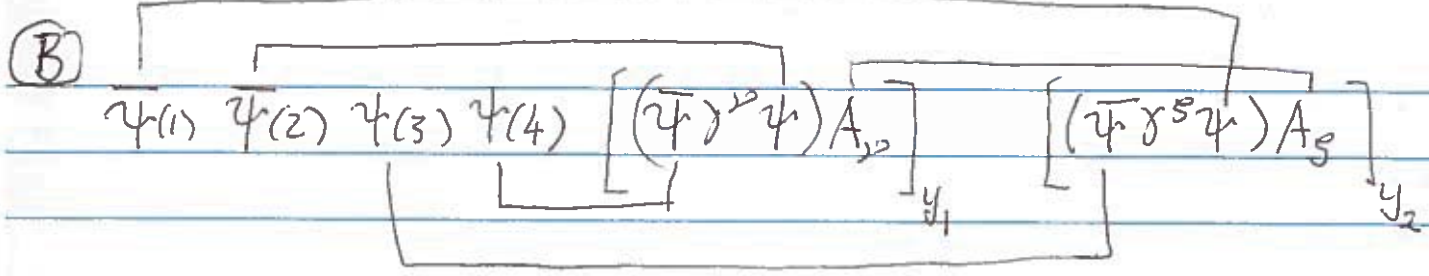
There are two distinct diagrams (not including $y_1 \leftrightarrow y_2$)

Ⓐ
$$\bar{\psi}(1) \bar{\psi}(2) \psi(3) \psi(4) \left[(\bar{\psi} \gamma^\mu \psi) A_\mu \right]_{y_1} \left[(\bar{\psi} \gamma^\nu \psi) A_\nu \right]_{y_2}$$

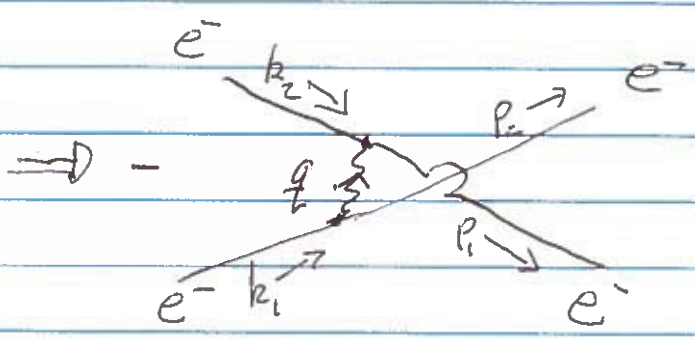
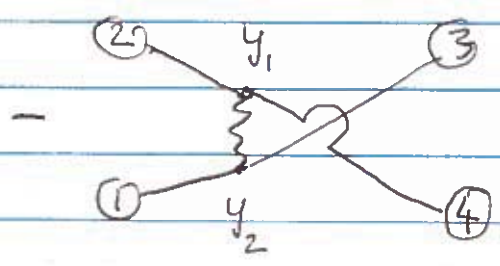
The diagram shows two interaction vertices, each represented by a square box. The left vertex is labeled y_1 and contains the expression $(\bar{\psi} \gamma^\mu \psi) A_\mu$. The right vertex is labeled y_2 and contains $(\bar{\psi} \gamma^\nu \psi) A_\nu$. Four external lines extend from the vertices: two from y_1 and two from y_2 . Lines connect the vertices: a top line connects the top of y_1 to the top of y_2 , a bottom line connects the bottom of y_1 to the bottom of y_2 , and two vertical lines connect the left and right sides of the two boxes.

Contract in order ② (+), then ① (-), then ④ (-) then ③ (+) \Rightarrow total sign $\equiv \boxed{+1}$





Contract in order ②(+), then ①(-), then ④(+), then ③(+)
 \Rightarrow total sign = $\boxed{-1}$



$$q = k_1 - p_2 = p_1 - k_2$$

$$iM_{fi} = (-ie)^2 \left\{ (\bar{u}(p_1, s_1') \gamma^\nu u(k_1, s_1)) (\bar{u}(p_2, s_2') \gamma_\nu u(k_2, s_2)) \right\}^* \frac{-i}{(k_1 - p_1)^2 + i\epsilon}$$

$$- (\bar{u}(p_1, s_1') \gamma^5 u(k_2, s_2)) (\bar{u}(p_2, s_2') \gamma_5 u(k_1, s_1)) \frac{-i}{(k_1 - p_2)^2 + i\epsilon}$$

$$2. \quad d\sigma = \frac{1}{4} \sum_{\text{spins}} |M_{fi}|^2 \frac{1}{4 \sqrt{(k_1 \cdot k_2)^2 - m_1^2 m_2^2}} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_1 - p_2) *$$

$$* \left(\prod_{j=1,2} \frac{d^3 p_j}{(2\pi)^3 2E_{p_j}} \right)$$

$$\overline{|M_{fi}|^2} = \frac{1}{4} \sum_{\text{spins}} |M_{fi}|^2 =$$

$$= \frac{e^4}{4} \sum_{s_1, s_2, s'_1, s'_2} \left\{ \left[\frac{1}{(k_1 - p_1)^4} \left| \left(\bar{u}(p_1, s'_1) \gamma^\nu u(k_1, s_1) \right) \left(\bar{u}(p_2, s'_2) \gamma_\nu u(k_2, s_2) \right) \right|^2 \right. \right. \\ \left. \left. + \left[(p_1, s'_1) \leftrightarrow (p_2, s'_2) \right] \right. \right. \\ \left. \left. - \left[\frac{1}{(k_1 - p_1)^2 (k_1 - p_2)^2} \left(\bar{u}(p_1, s'_1) \gamma^\nu u(k_1, s_1) \right) \left(\bar{u}(p_2, s'_2) \gamma_\nu u(k_2, s_2) \right) \right. \right. \right. \right. \\ \left. \left. \left. + \left(\bar{u}(k_2, s_2) \gamma^\nu u(p_1, s'_1) \right) \left(\bar{u}(k_1, s_1) \gamma_\nu u(p_2, s'_2) \right) \right] + \text{h.c.} \right] \right\}$$

The first 2 terms inside $\{ \}$ are similar to what we had for $e^+ e^- \rightarrow \mu^+ \mu^-$. Using

$$\sum_s u^\dagger(p, s) u(p, s) = (\not{p} + m_e)_{\text{tr}} \quad \text{and ignoring } m_e$$

$$\text{First 2 terms} = \frac{e^4}{4} \left\{ \frac{1}{(k_1 - p_1)^4} \text{tr} \{ \not{p}_1 \gamma^\nu \not{k}_1 \gamma^\nu \} \cdot \text{tr} \{ \not{p}_2 \gamma_\nu \not{k}_2 \gamma_\nu \} \right. \\ \left. + \frac{1}{(k_1 - p_2)^4} \text{tr} \{ \not{p}_2 \gamma^\nu \not{k}_1 \gamma^\nu \} \cdot \text{tr} \{ \not{p}_1 \gamma_\nu \not{k}_2 \gamma_\nu \} \right\}$$

Similarly the third (cross) term gives

$$\underline{\text{cross term}} = \frac{e^4}{4} \frac{-1}{(k_1 - p_1)^2 (k_1 - p_2)^2} \left[t_2 \{ \not{p}_1 \gamma^\mu \not{k}_1 \gamma_\mu \not{p}_2 \gamma_\nu \not{k}_2 \gamma^\nu \} + \text{h.c.} \right]$$

$$\text{Use } t_2 \{ \not{p}_1 \gamma^\mu \not{k}_1 \gamma^\nu \} = 4 \left[p_1^\mu k_1^\nu + p_1^\nu k_1^\mu - (p_1 \cdot k_1) g^{\mu\nu} \right]$$

$$\left[t_2 \{ \not{p}_2 \gamma_\mu \not{k}_2 \gamma_\nu \} = 4 \left[p_{2\mu} k_{2\nu} + p_{2\nu} k_{2\mu} - (p_2 \cdot k_2) g_{\mu\nu} \right] \right]$$

$$\rightarrow t_2 \{ \not{p}_1 \gamma^\mu \not{k}_1 \gamma^\nu \} \cdot t_2 \{ \not{p}_2 \gamma_\mu \not{k}_2 \gamma_\nu \} = 32 \left[(p_1 \cdot p_2) (k_1 \cdot k_2) + (p_1 \cdot k_2) (k_1 \cdot p_2) \right]$$

Also from $p_1 \rightleftharpoons p_2$

$$\left[t_2 \{ \not{p}_2 \gamma^\mu \not{k}_1 \gamma^\nu \} \cdot t_2 \{ \not{p}_1 \gamma_\mu \not{k}_2 \gamma_\nu \} = 32 \left[(p_1 \cdot p_2) (k_1 \cdot k_2) + (p_2 \cdot k_2) (k_1 \cdot p_1) \right] \right]$$

For the "cross term" use H.L.M. eq (6.24) repeatedly

$$t_2 \{ \not{p}_1 \gamma^\mu \not{k}_1 \gamma_\mu \not{p}_2 \gamma_\nu \not{k}_2 \gamma^\nu \} =$$

$$= -2 t_2 \{ \not{p}_1 \not{p}_2 \gamma_\mu \not{k}_1 \not{k}_2 \gamma^\mu \} = -8 (k_1 \cdot k_2) t_2 \{ \not{p}_1 \not{p}_2 \}$$

$$= -32 \left[(k_1 \cdot k_2) (p_1 \cdot p_2) \right]$$

$$\overline{|M_{fi}|^2} = e^4 \left\{ \frac{8}{(k_1 - p_1)^4} \left[(p_1 \cdot p_2) (k_1 \cdot k_2) + (p_1 \cdot k_2) (k_1 \cdot p_2) \right] \right.$$

$$+ \frac{8}{(k_1 - p_2)^4} \left[(p_1 \cdot p_2) (k_1 \cdot k_2) + (p_2 \cdot k_2) (k_1 \cdot p_1) \right]$$

$$\left. + \frac{16}{(k_1 - p_1)^2 (k_1 - p_2)^2} \left[(p_1 \cdot p_2) (k_1 \cdot k_2) \right] \right\}$$

In CM one has:

$$k_1 : (E, \vec{k}) , k_2 : (E, -\vec{k})$$

$$P_1 : (E', \vec{P}) , P_2 : (E', -\vec{P})$$

$$(k_1 \cdot k_2) = E^2 + \vec{k}^2 \approx 2E^2$$

$$(P_1 \cdot P_2) \approx 2E'^2$$

$$(k_1 \cdot P_1) = EE' (1 - \cos\theta) = 2EE' \sin^2 \frac{\theta}{2} = (k_2 \cdot P_2)$$

$$(k_1 \cdot P_2) = EE' (1 + \cos\theta) = 2EE' \cos^2 \frac{\theta}{2} = (k_2 \cdot P_1)$$

$$(k_1 - P_1)^2 = -2(k_1 \cdot P_1) = -4EE' \sin^2 \frac{\theta}{2}$$

$$(k_1 - P_2)^2 = -2(k_1 \cdot P_2) = -4EE' \cos^2 \frac{\theta}{2}$$

One also has

$$\int d^3P_1 \int d^3P_2 \delta^{(4)}(k_1 + k_2 - P_1 - P_2) \rightarrow \int d^3P \delta(2E - 2E')$$

$$\sigma = \int \frac{d^3P}{(2\pi)^2} \frac{1}{4(2E^2)} \frac{1}{(2E')^2} \delta(2E - 2E') |M_{fi}|^2$$

$$= \int \frac{d\Omega}{64E^2} \frac{1}{(2\pi)^2} |M_{fi}|^2 \Big|_{E'=E}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64E^2} \frac{1}{(2\pi)^2} |M_{fi}|^2 \Big|_{E'=E}$$

$$\begin{aligned}
 \overline{|M_{fi}|^2} \Big|_{E'=E} &= e^4 \left\{ \frac{8}{16E^4 \sin^4 \frac{\theta}{2}} \left[(2E^2)^2 + (2E^2)^2 \cos^4 \frac{\theta}{2} \right] \right. \\
 &+ \frac{8}{16E^4 \cos^4 \frac{\theta}{2}} \left[(2E^2)^2 + (2E^2)^2 \sin^4 \frac{\theta}{2} \right] \\
 &\left. + \frac{16}{16E^4 \cos^2 \frac{\theta}{2} \cdot \sin^2 \frac{\theta}{2}} (2E^2)^2 \right\}
 \end{aligned}$$

Let $\alpha \equiv \frac{e^2}{4\pi}$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2} \left\{ \frac{(1 + \cos^4 \frac{\theta}{2})}{\sin^4 \frac{\theta}{2}} + \frac{(1 + \sin^4 \frac{\theta}{2})}{\cos^4 \frac{\theta}{2}} + \frac{2}{\sin^2 \frac{\theta}{2} \cdot \cos^2 \frac{\theta}{2}} \right\}$$