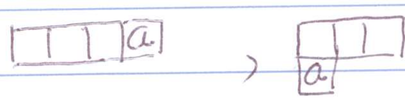
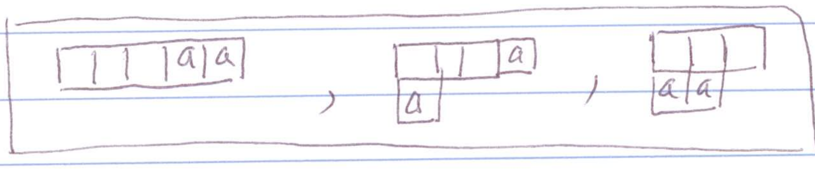
P8802.01 Set #4 Solutions

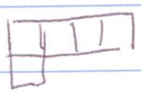
1.(a)

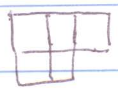
$6 \otimes 10$: 

1st a : 

2nd a :  (1)

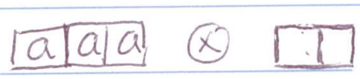
 : $(p,q) = (5,0)$ $\dim = \frac{1}{2} 6 \cdot 1 \cdot 7 = 21$

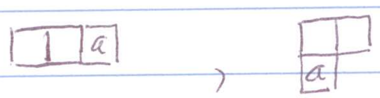
 : $(3,1)$ $\dim = \frac{1}{2} 4 \cdot 2 \cdot 6 = 24$

 : $(1,2)$ $\dim = \frac{1}{2} 2 \cdot 3 \cdot 5 = 15$

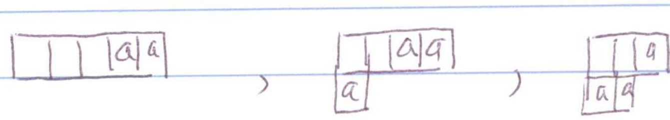
$\Rightarrow \boxed{6 \otimes 10 = 21 \oplus 24 \oplus 15}$

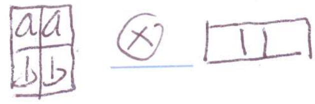
LHS : $10 \cdot 6 = 60$, RHS : $21 + 24 + 15 = 60$

$10 \otimes 6$: 

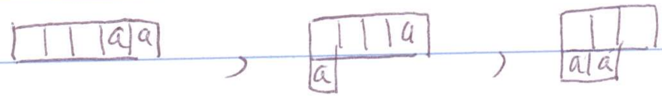
1st a : 

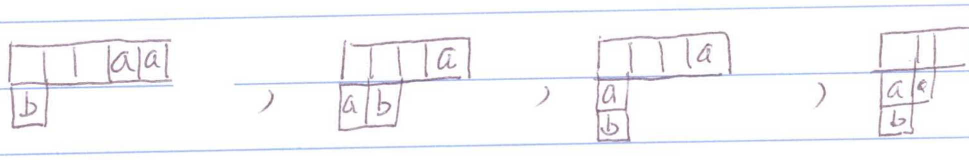
2nd a : 

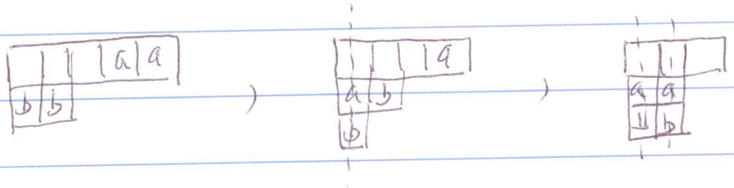
3rd a :  ⇐ same set as (1)

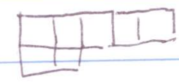
1.(b) $\bar{6} \otimes 10$: 

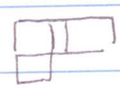
1^{st} & 2^{nd} $\boxed{a}$: same as (1)



1^{st} $\boxed{b}$: 

2^{nd} $\boxed{b}$:  (2)

 : $(3,2)$ $\dim = \frac{1}{2} 4 \cdot 3 \cdot 7 = 42$

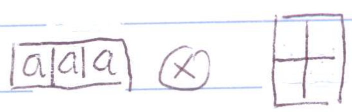
 : $(2,1)$ $\dim = \frac{1}{2} 3 \cdot 2 \cdot 5 = 15$

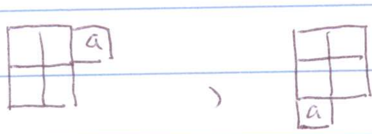
 $(1,0)$ $\dim = 3$

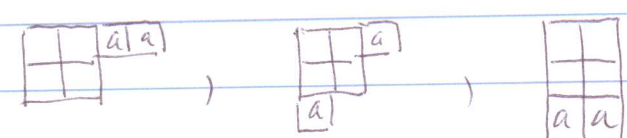
$$\Rightarrow \boxed{\bar{6} \otimes 10 = 42 \oplus 15 \oplus 3}$$

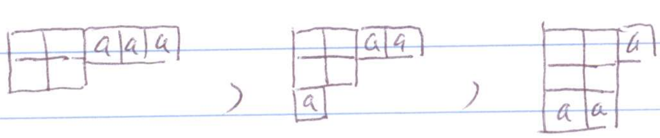
LHS : $10 \cdot 6 = 60$

RHS : $42 + 15 + 3 = 60$

$10 \otimes \bar{6}$: 

1^{st} $\boxed{a}$: 

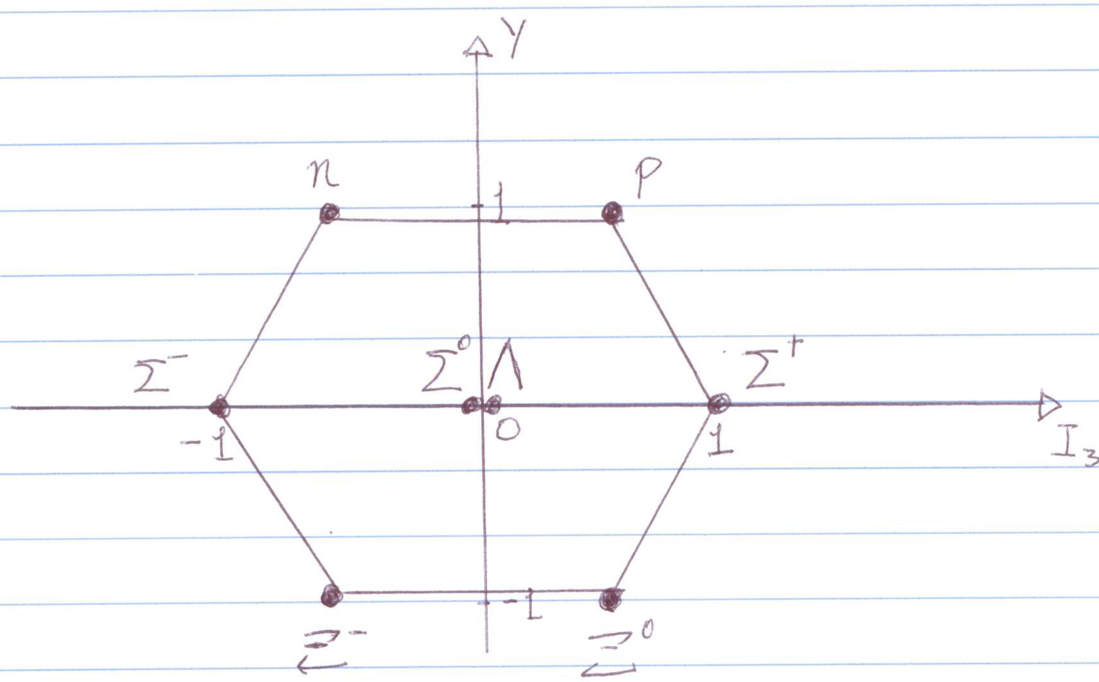
2^{nd} $\boxed{a}$: 

3^{rd} $\boxed{a}$:  $\Leftarrow$ same as (2)

2. (a)

Baryon	Quark Content	I_3	S	B	Y	Q
n	ddu	$-1/2$	0	1	1	0
p	uud	$1/2$	0	1	1	1
Σ^-	dds	-1	-1	1	0	-1
Σ^0	uds	0	-1	1	0	0
Σ^+	uus	$+1$	-1	1	0	1
Λ	uds	0	-1	1	0	0
Ξ^-	dss	$-1/2$	-2	1	-1	-1
Ξ^0	uss	$1/2$	-2	1	-1	0

1. (b)



3.

In the Quark Model one views a quark - antiquark boundstate very much like a central potential 2-body problem in Quantum Mechanics (e.g. the Hydrogen Atom).
Let $\vec{r}_q$, $\vec{r}_{\bar{q}}$ denote the position for the quark and antiquark and $\vec{r} = \vec{r}_q - \vec{r}_{\bar{q}}$.
The space part of the wave function can be written as $R_n(r)\ Y_{LM}(\theta,\varphi)$
Under a parity operation $\hat{P}$ one has

$$\vec{r} \xrightarrow{\hat{P}} -\vec{r} \quad \text{or} \quad (r,\theta,\varphi) \xrightarrow{\hat{P}} (r, \pi-\theta, \varphi+\pi)$$

For the spherical harmonics this implies

$$Y_{LM}(\theta,\varphi) \xrightarrow{\hat{P}} (-1)^L\, Y_{LM}(\theta,\varphi)$$

So, the full wave fcn. obeys

$$\psi_{q\bar{q}} \xrightarrow{\hat{P}} P_q \cdot P_{\bar{q}}\ (-1)^L\ \psi_{q\bar{q}}$$

P_q and $P_{\bar{q}}$ denote intrinsic parity of the quark or antiquark respectively.
From our solutions to the Dirac Eq. and
$\psi_D(\vec{r}) \to \gamma_0\, \psi_D(-\vec{r})$, $\vec{p} \to -\vec{p}$ one sees that $P_q \cdot P_{\bar{q}} = -1$

$\Rightarrow$ Halzen & Martin Eq (2.51)

$$\boxed{\psi_{q\bar{q}} \xrightarrow{\hat{P}} -(-1)^L\, \psi_{q\bar{q}}}$$

The charge conjugation operation $\hat{C}$ means particle-antiparticle interchange.
This brings in 3 factors.

(i) interchange of 2 fermions $\Rightarrow (-1)$

(ii) $\vec{r}_q \rightleftarrows \vec{r}_{\bar{q}} \Rightarrow (-1)^L$ (see discussion of parity)

(iii) interchange of spins $\Rightarrow (-1)^{S+1} = \begin{cases} -1 & S=0 \text{ i.e. antisymmetric spin wave fcn} \\ +1 & S=1 \text{ symmetric spin wave fcn.} \end{cases}$

Putting these factors together

$$\boxed{\psi_{q\bar{q}} \xrightarrow{\hat{C}} (-1)^{S+L}\, \psi_{q\bar{q}}}$$

i.e. H. & M. Eq. [2.52]

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4 (a)

In writing ${}^1S_0(\eta_c)$ or ${}^3S_1(J/\psi)$ one is using the "spectroscopic" notation familiar from atomic physics

$$^{(2S+1)}\mathcal{L}_J$$

S, P, D, ... for $L = 0, 1, 2, \cdots$

So the η_c has $S=0$, $L=0$, $J=0$

and hence $\boxed{J^{PC}(\eta_c) = 0^{-+}}$

For J/ψ charmonium one has $S=1$, $L=0$, $J=1$
and $\boxed{J^{PC}(J/\psi) = 1^{--}}$

4.
Combining spins of the heavy quark and light antiquark
the total $S = 0$ or 1
The orbital angular momentum will increase $(L=0, 1, \cdots)$
as one goes from ground to excited states.
Finally combining S & L $\Rightarrow$ possible J values

J^P	0^-	1^-	0^+	1^+		2^+
(S,L)	$(0,0)$	$(1,0)$	$(1,1)$	$(0,1)$	$(1,1)$	$(1,1)$
	$L=0$ S-states		$L=1$ P-states			

"C" is not a relevant quantum number here
since B-mesons ($b\bar{q}$ bound states) are not
eigenstates of $\hat{C}$.