Elastic vs Inelastic Electron-Proton Scattering

- Elastic collision: In the previous lecture we discuss the scattering reaction:
  \[ e^-p \rightarrow e^-p \]
  - The reaction could be described by the Rosenbluth formula:
  \[
  \frac{d\sigma}{d\Omega} = \left( \frac{\alpha \hbar}{4M_p E \sin^2(\theta/2)} \right)^2 \frac{E'}{E} \left[ 2K_1(q^2)\sin^2(\theta/2) + K_2(q^2)\cos^2(\theta/2) \right]
  \]
  - Form factors \( K_1 \) and \( K_2 \) contain information about the structure and size of the proton.
  - This process is an example of an elastic scattering:
    - same kind and number of particles in the initial and final state.
      - no new particles are created in the collision
      - satisfy the classical definition of an elastic collision:
        initial kinetic energy = final kinetic energy.

- Inelastic collision: “new” particles in the final state, e.g.:
  \[ e^-p \rightarrow e^-p\pi^0 \]
  \[ e^-p \rightarrow e^-\Delta^+ \]
  \[ e^-p \rightarrow e^-pK^+K^- \]
  - These collisions are mediated by photons for center of mass energies well below the Z-boson mass.
    - Classified as electromagnetic interactions.
    - All quantum numbers respected by the electromagnetic interaction must be conserved.
Inelastic ep Scattering

- There are many final states in an inelastic scattering.
  - It is convenient to define a quantity called inclusive cross section.
  - Here we are interested in the reaction:
    \[ e^p \rightarrow e^X^+ \]
  - It called an inclusive reaction because we don't measure any of the properties of “\(X\).”
  - We include all available final states.
  - Experimentally: we only measure the 4-vector (energy and angle) of the final state electron.
  - The inclusive cross section (in the lab frame) for \(e^p \rightarrow e^X^+\) can be written as:

\[
\frac{d\sigma}{dE'd\Omega} = \left( \frac{\alpha \hbar}{2E \sin^2 (\theta/2)} \right)^2 \left[ 2W_1 \sin^2 (\theta/2) + W_2 \cos^2 (\theta/2) \right]
\]

- This is similar in form to the Rosenbluth formula.
- There are some important differences:
  - This is the cross section for the scattered electron to have energy \(E'\) within a solid angle \(d\Omega\).
  - The Rosenbluth formula does not contain this dependence.
  - Elastic scattering: energy of the scattered electron \((E')\) is determined by the scattering angle \(\theta\).
  - Inelastic scattering: energy of the scattered electron \((E')\) is not uniquely determined by \(\theta\).
The scattered electron must be described (kinematically) by two variables.

- A common set of (Lorentz invariant) kinematic variables are:
  
  \[ q^2 \quad \text{and} \quad x = \frac{-q^2}{2qp} \]

  - \( q^2 < 0 \) and \( 0 \leq x \leq 1 \)
  - \( p \): 4-vector of the target proton (e.g. \((M,0,0,0)\) in “lab” frame)
  - \( q \): difference between the incoming electron (beam) and outgoing (scattered) electron:
    \[ q \equiv p_e - p_e' = p_X - p_T \]

  \[ q^2 = (p_X - p_T)^2 \]

  \[ = (E_X - M)^2 - \vec{p}_X^2 \]

  \[ = M_X^2 + M^2 - 2ME_X \]

  \[ 2pq = 2M(E_X - M) - 2 \cdot 0 \cdot (\vec{p}_X - \vec{p}_T) \]

  \[ = 2M(E_X - M) \]

  - It is convenient to use the positive variable \( Q^2 \):
    \[ Q^2 = -q^2 = (\vec{p}_e - \vec{p}_e')^2 - (E_e - E'_e)^2 \]

    - This is just the square of the invariant mass of the virtual photon.
    - In the limit \( E_e & E'_e \gg m_e \)
      \[ Q^2 = 2E_eE'_e - 2\vec{p}_e\vec{p}_e' \]

      \[ = 2E_eE'_e(1 - \cos \theta) \]

  - For elastic scattering \( e^- p \rightarrow e^- p \): \( M_X = M \)

\[ x = \frac{-q^2}{2pq} = \frac{2M_X^2 - 2ME_X}{2M(E_X - M)} = 1 \]
Looking Inside a Nucleon

- What if spin $\frac{1}{2}$ point-like objects are inside the proton?
  - As $Q^2$ increases, the wavelength of the virtual photon decreases.
  - At some point we should be able to see “inside” the proton.
  - This situation was analyzed by many people in the late 1960’s (Bjorken, Feynman, Callan and Gross).
  - Predictions were made for spin 1/2 point-like objects inside the proton (or neutron).
  - These theoretical predictions were quickly verified by a new generation of electron scattering experiments performed at SLAC and elsewhere (DESY, Cornell…)
- Re-write $W_1$ and $W_2$ in terms of $F_1$ and $F_2$:
  $$\frac{Q^2}{2Mx} W_2(Q^2,x) \rightarrow F_2(x)$$
  $$MW_1(Q^2,x) \rightarrow F_1(x)$$
- In the limit where $Q^2$ and $2pq$ are large,
  - Bjorken predicted that the form factors would only depend on the scaling variable $x$:
    $$x = \frac{Q^2}{2pq} \text{ Bjorken Scaling}$$
  - These relationships are a consequence of point-like objects (small compared to the wavelength of the virtual photon) being “inside” the proton or neutron.
Looking Inside a Nucleon

- Callan-Gross Relationship:
  - If the point-like objects are spin $\frac{1}{2}$
    - Callan and Gross predicted that the two form factors would be related:
      \[ 2x F_1(x) = F_2(x) \]
    - If however the objects had spin 0, then they predicted:
      \[ 2x F_1(x)/F_2(x) = 0 \]
  - But are these objects quarks?
    - Do they have fractional charge?
    - In the late 1960’s most theorists called these point-like spin $\frac{1}{2}$ objects “partons”.

Good agreement with spin $\frac{1}{2}$ point-like objects inside proton or neutron!
Are Partons Quarks?

- Do these objects have fractional electric charge?
  \[ \frac{2}{3} |e| \text{ or } -\frac{1}{3} |e| \]
- First we have to realize that \( x \) represents the fraction of the proton’s momentum carried by struck quark.
- Next, we take into account how the quarks share the nucleon’s momentum.
  - A quark inside a nucleon now have some probability \( (P) \) distribution to have momentum fraction \( x \):
    \[ f(x) = \frac{dP}{dx} \]
    - probability to find a quark with momentum fraction \( x \) to \( dx \)
  - Structure function for inelastic scattering of electron off a quark of momentum fraction \( x \) and charge \( e_i \):
    \[ F_1 = \frac{e_i^2 f(x)}{2} \text{ and } F_2 = e_i^2 x f(x) \]
- To get the structure function of a proton (or neutron) we must add all the quark contributions.
  - Let \( u(x) \) denote the \( x \) probability distribution for an up quark etc.:
    \[ u_p(x) = \text{pdf for up quark in a proton} \]
    \[ u_n(x) = \text{pdf for up quark in a neutron} \]
    \[ d_p(x) = \text{pdf for down quark in a proton} \]
    \[ d_n(x) = \text{pdf for down quark in a neutron} \]
- The structure functions:
  \[ F_{1p} = \frac{1}{2} \sum_{i=1}^{2} e_i^2 f_i(x) = \frac{1}{2} \left[ \left( \frac{2}{3} \right)^2 u_p(x) + \left( \frac{-1}{3} \right)^2 d_p(x) \right] \]
  \[ F_{2p} = x \sum_{i=1}^{2} e_i^2 f_i(x) = x \left[ \left( \frac{2}{3} \right)^2 u_p(x) + \left( \frac{-1}{3} \right)^2 d_p(x) \right] \]
  \[ F_{1n} = \frac{1}{2} \sum_{i=1}^{2} e_i^2 f_i(x) = \frac{1}{2} \left[ \left( \frac{-1}{3} \right)^2 d_n(x) + \left( \frac{2}{3} \right)^2 u_n(x) \right] \]
  \[ F_{2n} = x \sum_{i=1}^{2} e_i^2 f_i(x) = x \left[ \left( \frac{2}{3} \right)^2 u_n(x) + \left( \frac{-1}{3} \right)^2 d_n(x) \right] \]
The simple model of having only three quarks in a proton does not describe the data! There must be a sea of quark-antiquark pairs.
Evidence for Sea Quarks

- We now talk about two types of quarks, valence and sea.
  - The valence quarks are the ones that we expect to be in the nucleon.
  - The sea quarks are the ones we get from the quark anti-quark pairs.
- A proton has two valence $u$ quarks and one valence $d$ quark.

- Generalize structure functions to allow quark anti-quark pairs to also exist (for a short time) in a proton:

\[
F_{1p} = \frac{1}{2} \left[ \left( \frac{2}{3} \right)^2 \left[ u_p(x) - \bar{u}_p(x) \right] + \left( \frac{-1}{3} \right)^2 \left[ d_p(x) - \bar{d}_p(x) + s_p(x) - \bar{s}_p(x) \right] \right]
\]

- By isospin invariance (up quark = down quark):
  - $u = u_p = d_n$
  - $d = d_p = u_n$

- If only “sea” quarks in proton and neutron:

\[
\frac{F_{2N}}{F_{2p}} = 1
\]
Electron vs Neutrino Scattering with Nucleons

- Compare electron-nucleon scattering with neutrino-nucleon scattering:
  - EM interaction (photon) sensitive to quark electric charge.
  - Neutrino scattering via $W$ exchange is blind to quark electric charge.
    - $W^+$ interacts with the down quark only.
  - The structure functions for $e(p+n)$ and $\nu(p+n)$ scattering:
    
    $$F_2^{ep+en}(x) = \left\{ \left( \frac{2}{3} \right)^2 xu + \left( \frac{-1}{3} \right)^2 xd \right\} + \left\{ \left( \frac{2}{3} \right)^2 xd + \left( \frac{-1}{3} \right)^2 xu \right\} = \frac{5}{9} \{xu(x) + xd(x)\}$$
    
    $$F_2^{vp+vn}(x) = 2xd(x) + 2xu(x)$$
    
    $$F_2^{vp+vn} = \frac{18}{5} F_2^{ep+en}$$
  - Can also measure the average fraction of the nucleon momentum carried by the quarks:
    - Integrate the area under the measured $\nu N$ curve:
      
      $$\frac{1}{0} \int F_2^{\nu N} dx \approx 0.5$$
    - Something else in the nucleon is carrying half of its momentum!
    - Gluons!

Good Agreement with prediction!
The parton charges are the quark charges.

K.K. Gan

L9: Partons