Transistors and Amplifiers

Hybrid Transistor Model for Small AC Signals

The previous model for a transistor used one parameter (\( h \), the current gain) to describe the transistor. This model is naive and doesn't explain many of the features of the three common forms of transistor amplifiers (common emitter etc.) For example, we could not calculate the output impedance of the common emitter amp with the one parameter model.

Very often in electronics we describe complex circuits in terms of an equivalent circuit or model. For the transistor we wish to have a model that relates the input currents and voltages to the output currents and voltages. We also wish this model to be linear in the currents and voltages. For a transistor this condition of linearity is true for small signals.

The most general linear model of the transistor is a 4-terminal "black box".

In this model we assume that the transistor is biased properly and often we do not even show the biasing circuit.

For the transistor where there are only 3 legs, one of the terminals is common between the input and output.

\[
\begin{align*}
&I_i \rightarrow \quad T \quad \leftarrow I_o \\
&V_i \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
The partial derivatives are called the hybrid (or \( h \)) parameters. We rewrite the above equations as:

\[
\begin{align*}
\frac{dV_i}{dt} &= h_{ii} \frac{di}{dt} + h_{i0} \frac{dV_o}{dt} \\
\frac{dI_o}{dt} &= h_{o0} \frac{di}{dt} + h_{o1} \frac{dV_o}{dt}
\end{align*}
\]

The parameters \( h_{oi} \) and \( h_{io} \) are unitless while \( h_{oo} \) has units \( W^{-1} \) (mhos) and \( h_{ii} \) has units \( W \).

The four \( h \) parameters are easily measured. For example to measure \( h_{ii} \) hold \( V_o \) (the output voltage) constant and measure \( V_{in}/I_{in} \). Unfortunately the \( h \) parameters are not constant. For example Figs. 11-14 of the 2N3904 spec sheet show the variation of the four parameters with collector current (\( I_C \)).

There are 3 sets of the 4 hybrid parameters, one for each type of amp (common emitter, common base, common collector). In order to differentiate one set of parameters from another the following notation is used:

**First subscript**
- \( i = \) input impedance
- \( o = \) output admittance
- \( r = \) reverse voltage ratio
- \( f = \) forward current ratio

**Second subscript**
- \( e = \) common emitter
- \( b = \) common base
- \( c = \) common collector

Thus for a common emitter amplifier we would write:

\[
\begin{align*}
\frac{dV_i}{dt} &= h_{ie} \frac{di}{dt} + h_{re} \frac{dV_o}{dt} \\
\frac{dI_o}{dt} &= h_{oe} \frac{di}{dt} + h_{o1} \frac{dV_o}{dt}
\end{align*}
\]

Typical values for the \( h \) parameters for a 2N3904 transistor in the common emitter configuration with \( I_C = 1 \) mA are as follows:

\[
\begin{align*}
h_{fe} &= 120, \quad h_{oe} = 8.7 \times 10^{-6} \, \Omega^{-1}, \quad h_{ic} = 3700 \, \Omega, \quad h_{re} = 1.3 \times 10^{-4}
\end{align*}
\]

The equivalent circuit for a transistor in the common emitter configuration looks like:

The circle is a voltage source, i.e. the voltage across this element is always equal to \( h_{re} V_o \) independent of the current through it. The triangle is a current source, the current through this element is always \( h_{fe} I_{in} \) independent of the voltage across the device.
We can calculate the voltage and current gain, and the input and output impedance of a common emitter amp using this model. Below is the equivalent circuit (without biasing network) for a CE amp attached to a voltage source (with resistance $R_s$) and load resistor ($R_{load}$).

**Current gain:** The current gain ($G_I$) is defined as: $G_I = I_o/I_{in}$.

Using Kirchhoff’s current law at the output side we have:

$$h_{fe}I_{in} + V_o h_{oe} = I_o$$

Using Kirchhoff’s voltage rule at the output we have:

$$V_o = h_{fe} I_{in}$$

Putting the two equations together we get:

$$G_I = I_o / I_{in} = h_{fe} / (1 + h_{oe} R_{load})$$

For typical CE amps, $h_{oe} R_{load} \ll 1$ and the gain reduces to familiar form:

$$G_I \approx h_{fe} / r_{BE}$$

**Voltage gain:** The voltage gain ($G_V$) is defined as: $G_V = V_o/V_{in}$.

This gain can be derived in a similar fashion as the current gain. The result is:

$$G_V = V_o / V_{in} = [h_{fe} R_{load}] / (D R_{load} + h_{ie})$$

with $D = h_{ie} h_{oe} h_{fe} / 10$

This reduces to a familiar form for most cases where $D R_{load} \ll h_{ie}$

$$G_V = h_{fe} R_{load} / h_{fe} = D R_{load} / r_{BE}$$

**Input Impedance:** The input impedance ($Z_i$) is defined as: $Z_i = V_{in}/I_{in}$.

$$Z_i = (D R_{load} + h_{ie}) / (1 + h_{oe} R_{load})$$

This reduces to a familiar form for most cases where $D R_{load} \ll h_{ie}$ and $h_{oe} R_{load} \ll 1$

$$Z_i = h_{ie} = h_{fe} r_{BE}$$

**Output Impedance:** The output impedance ($Z_o$) is defined as: $Z_o = V_o/I_o$.

$$Z_o = (R_s + h_{ie}) / (D + h_{oe} R_s)$$

$Z_o$ does not reduce to a simple expression. As the denominator is small, $Z_o$ is as advertised large.
Feedback and Amplifiers

• Consider the following common emitter amplifier:

This amp differs slightly from the CE amp we saw before in that the bias resistor $R_2$ is connected to the collector resistor $R_1$ instead of directly to $V_{cc}$.

• How does this effect $V_{out}$?
  a) If $V_{out}$ decreases (moves away from $V_{cc}$) then $I_2$ increases which means that $V_B$ decreases (gets closer to ground). However, if $V_B$ decreases, then $V_{out}$ will increase since $\Delta V_{out} = -\Delta V_B R_1/R_E$.
  b) If $V_{out}$ increases (moves towards $V_{cc}$) then $I_2$ decreases which means that $V_B$ increases (moves away from ground). However, if $V_B$ increases, then $V_{out}$ will decrease since $\Delta V_{out} = -\Delta V_B R_1/R_E$.

This is an example of NEGATIVE FEEDBACK

Negative Feedback is good:
  - Stabilizes amplifier against oscillation
  - Increases the input impedance of the amplifier
  - Decreases the output impedance of the amplifier

Positive Feedback is bad:
  - Causes amplifiers to oscillate.
Feedback Fundamentals:

The above is a general feedback circuit. Without feedback the output and input are related by:

\[ V_{\text{out}} = AV_{\text{in}} \]

The feedback (the box with the B) returns a portion of the output voltage to the amplifier through the "mixer".

Note: The feedback network on the AM radio is the collector to base resistors (R_3, R_5)

The input to the amplifier is:

\[ V_x = V_{\text{in}} + B V_{\text{out}} \]

The gain with feedback is:

\[ V_{\text{out}} = AV_x = A(V_{\text{in}} + BV_{\text{out}}) \]

\[ G = V_{\text{out}} / V_{\text{in}} = A / (1 - AB) \]

Some definitions:

- \( A \) is the open loop gain, \( AB \) is the loop gain, \( G \) is the closed loop gain.

Let's define \( A > 0 \) (positive). Then there are two cases to consider depending on whether or not \( AB \) is positive or negative.

1) \( AB > 0 \). This is positive feedback. As \( AB \rightarrow 1 \), \( G \rightarrow \infty \). The circuit is unstable and oscillates if \( AB = 1 \).

2) \( AB < 0 \). This is negative feedback. As \( A \rightarrow \infty \), an amazing thing happens:

\( |AB| \rightarrow 0 \), \( |G| \Rightarrow 1/|B| \).

**For large amplifier gain (A) the circuit properties are determined by the feedback loop.**

For example: \( A = 10^5 \) and \( B = -0.01 \) then \( G = 100 \).

The stability of the gain is also determined by the feedback loop (B) and not the amplifier (A).

For example if \( B \) is held fixed at 0.01 and \( A \) varies:

<table>
<thead>
<tr>
<th>( A )</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5 \times 10^3 )</td>
<td>98.3</td>
</tr>
<tr>
<td>( 1 \times 10^4 )</td>
<td>99.0</td>
</tr>
<tr>
<td>( 2 \times 10^4 )</td>
<td>99.6</td>
</tr>
</tbody>
</table>

Thus circuits can be made stable with respect to variations in the transistor characteristics as long as \( B \) is stable. \( B \) can be made from precision components such as resistors.
Operational Amplifiers (Op Amps)

- Op amps are very high gain ($A = 10^5$) differential amplifiers.

  A differential amp has two inputs ($V_1$, $V_2$) and output $V_{out} = A(V_1 - V_2)$ where $A$ is the amplifier gain.

  ![Non-inverting input](power connections not shown)

  ![Inverting input](power connections not shown)

  If an op amp is used without feedback and $V_1 \neq V_2$, then $V_{out}$ saturates at the power supply voltage (either positive or negative supply).

- Op amps are almost always used with negative feedback. The output is connected to the (inverting) input.

- Op amps come in "chip" form. They are made up of complex circuits with 20-100 transistors.

<table>
<thead>
<tr>
<th>Ideal Op Amp</th>
<th>Real Op Amp [A741]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage gain (open loop)</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Input impedance</td>
<td>$\bullet$</td>
</tr>
<tr>
<td>Output impedance</td>
<td>0</td>
</tr>
<tr>
<td>Slew rate*</td>
<td>$\bullet$</td>
</tr>
<tr>
<td>Power consumption</td>
<td>0</td>
</tr>
<tr>
<td>$V_{out}$ with $V_{in} = 0$</td>
<td>0</td>
</tr>
<tr>
<td>Price</td>
<td>0$</td>
</tr>
</tbody>
</table>

  * slew rate is how fast the output can change

- When working with op amps using negative feedback the following two simple rules (almost) always apply:
  1) **No current goes into the op amp.**
  2) **Both input terminals of the op amp have the same voltage.**

  The first rule reflects the high input impedance of the op amp. The second rule has to do with the actual circuitry making up the op amp.

- Some examples of op amp circuits with negative feedback:

  **Voltage Follower**

  $V_{in}$ ![Non-inverting input](power connections not shown)

  $V_{out} = V_{in}$

  (power connections not shown)

  The feedback network is just a wire connecting the output to the input. By rule #2, the inverting (-) input is also at $V_{in}$. Thus $V_{out} = V_{in}$.

  What good is this circuit? It is mainly used as a buffer as it has high input impedance (MΩ) and low output impedance (100 Ω).
Inverting Amplifier:

\[
\begin{align*}
V_{in} & \rightarrow A \quad V_{out} \\
\frac{1}{R_1} & \quad R_f
\end{align*}
\]

(power connections not shown)

By rule #2, point A is at ground.

By Rule #1, no current is going into the op amp.

We can redraw the circuit as:

\[
\begin{align*}
V_{in} & \rightarrow I_{in} \rightarrow I_{out} \rightarrow V_{out} \\
\frac{1}{R_1} & \quad \frac{1}{R_f}
\end{align*}
\]

\[
V_{in}/R_1 + V_{out}/R_f = 0
\]

\[
V_{out}/V_{in} = \frac{R_f}{R_1}
\]

Thus the closed loop gain is \(R_f/R_1\). The minus sign in the gain means that the output has the opposite polarity as the input. See page 8 for more details.

Non-Inverting Amplifier:

\[
\begin{align*}
V_{in} & \rightarrow A \quad V_{out} \\
\frac{1}{R_1} & \quad R_f
\end{align*}
\]

(power connections not shown)

By rule #2, point A is \(V_{in}\).

By Rule #1, no current is going into the op amp. We can redraw the circuit as:

\[
\begin{align*}
I_{in} & \rightarrow V_{in} \rightarrow I_{out} \rightarrow V_{out} \\
\frac{1}{R_1} & \quad \frac{1}{R_f}
\end{align*}
\]

\[
V_{in}/R_1 + (V_{in} - V_{out})/R_f = 0
\]

\[
V_{out}/V_{in} = (R_1 + R_f)/R_1
\]

Thus the closed loop gain is \((R_1 + R_f)/R_1\). For this circuit the output has the same polarity as the input.
Integrating Amplifier:

\[ V_{\text{in}} \overset{R_1}{\rightarrow} V_A \overset{A}{\rightarrow} V_{\text{out}} \]

(power connections not shown)

Again, using the two rules for op amp circuits we redraw the circuit as:

\[ V_{\text{in}} \overset{R_1}{\rightarrow} \overset{C}{\rightarrow} V_{\text{out}} \]

\[ \frac{V_{\text{in}}}{R_1} + C \frac{dV_{\text{out}}}{dt} = 0 \]

Remember \( Q = CV \) ! Solving the above for the voltage gain we obtain:

\[ V_{\text{out}} = \frac{0}{CR_1} \int V_{\text{in}} \, dt \]

Thus the output voltage is related to the integral of the input voltage. The negative sign in the gain means that \( V_{\text{in}} \) and \( V_{\text{out}} \) have opposite polarity.

Op Amps and Analog Calculations:

Op amps were invented before transistors to perform analog calculations. Their main function was to solve differential equations in real time. For example, suppose we wanted to solve the following:

\[ \frac{d^2 x}{dt^2} = g \]

This describes a body under constant acceleration (gravity if \( g = 9.8 \text{ m/s}^2 \)).

The following circuit gives an output which is the solution to the above differential equation:

The input voltage is a constant (\( = g \)). For convenience we pick \( RC = 1 \). At point A we have:

\[ V_A = \int \int V_{\text{in}} \, dt = \int \int \frac{d^2 x}{dt^2} \, dt = \int \frac{dx}{dt} \, dt \]

The output voltage (\( V_{\text{out}} \)) is the integral of \( V_A \):

\[ V_{\text{out}} = \int \int V_A \, dt = \int \int \frac{dx}{dt} \, dt = x(t) \]

If we want non-zero boundary conditions (e.g. \( V(t = 0) = 1 \text{ m/s} \)) we add a DC voltage at point A.