PROBLEMS

Complete solutions to all problems—except those with an asterisk—can be found in the back of the book.

2.1 How many “yellow” lightwaves ($\lambda = 580$ nm) will fit into a distance in space equal to the thickness of a piece of paper (0.003 in.)? How far will the same number of microwaves ($\nu = 10^{10}$ Hz, i.e., 10 GHz, and $v = 3 \times 10^8$ m/s) extend?

2.2* The speed of light in vacuum is approximately $3 \times 10^8$ m/s. Find the wavelength of red light having a frequency of $5 \times 10^{14}$ Hz. Compare this with the wavelength of a 60-Hz electromagnetic wave.

2.3* It is possible to generate ultrasonic waves in crystals with wavelengths similar to light ($5 \times 10^{-5}$ cm) but with lower frequencies ($6 \times 10^9$ Hz). Compute the corresponding speed of such a wave.

2.4* A youngster in a boat on a lake watches waves that seem to be an endless succession of identical crests passing with a half-second interval between each. If every disturbance takes 1.5 s to sweep straight along the length of her 4.5 m-long boat, what are the frequency, period, and wavelength of the waves?

2.5* A vibrating hammer strikes the end of a long metal rod in such a way that a periodic compression wave with a wavelength of 4.3 m travels down the rod’s length at a speed of 3.5 km/s. What was the frequency of the vibration?

2.6 A violin is submerged in a swimming pool at the wedding of two scuba divers. Given that the speed of compression waves in pure water is 1498 m/s, what is the wavelength of an A-note of 440 Hz played on the instrument?

2.7* A wavepulse travels 10 m along the length of a string in 2.0 s. A harmonic disturbance of wavelength 0.50 m is then generated on the string. What is its frequency?

2.8* Show that for a periodic wave $\omega = (2\pi/\lambda)\nu$.

2.9* Make up a table with columns headed by values of $\theta$ running from $-\pi/2$ to $2\pi$ in intervals of $\pi/4$. In each column place the corresponding value of $\sin \theta$, beneath those the values of $\cos \theta$, beneath those the values of $\sin (\theta - \pi/4)$, and similarly with the functions $\sin (\theta - \pi/2)$, $\sin (\theta - 3\pi/4)$, and $\sin (\theta + \pi/2)$. Plot each of these functions, noting the effect of the phase shift. Does $\sin \theta$ lead or lag $\sin (\theta - \pi/2)$. In other words, does one of the functions reach a particular magnitude at a smaller value of $\theta$ than the other and therefore lead the other (as $\cos \theta$ leads $\sin \theta$)?

2.10* Make up a table with columns headed by values of $kx$ running from $x = -\lambda/2$ to $x = +\lambda$ in intervals of $\lambda/4$—of course, $k = 2\pi/\lambda$. In each column place the corresponding values of $\cos (kx - \pi/4)$ and beneath that the values of $\cos (kx + 3\pi/4)$. Next plot the functions $15 \cos (kx - \pi/4)$ and $25 \cos (kx + 3\pi/4)$.

2.11* Make up a table with columns headed by values of $\omega t$ running from $t = -\pi/2$ to $t = +\pi$ in intervals of $\pi/4$—of course, $\omega = 2\pi/\tau$. In each column place the corresponding values of $\sin (\omega t + \pi/4)$ and $\sin (\pi/4 - \omega t)$ and then plot these two functions.

2.12* The profile of a transverse harmonic wave, traveling at 1.2 m/s on a string, is given by

$$y = (0.02 \text{ m}) \sin (157 \text{ m}^{-1} \times x)$$

Determine its amplitude, wavelength, frequency, and period.

2.13* Figure P.2.13 represents the profile ($t = 0$) of a transverse wave on a string traveling in the positive $x$-direction at a speed of 20.0 m/s. (a) Determine its wavelength. (b) What is the frequency of the wave? (c) Write down the wavefunction for the disturbance. (d) Notice that as the wave passes any fixed point on the $x$-axis the string at that location oscillates in time. Draw a graph of the $\psi$ versus $x$ showing how a point on the rope at $x = 0$ oscillates.

![Figure P.2.13](image-url)

2.14* Figure P.2.14 represents the profile ($t = 0$) of a transverse wave on a string traveling in the positive $z$-direction at a speed of 100 cm/s. (a) Determine its wavelength. (b) Notice that as the wave passes any fixed point on the $z$-axis the string at that location oscil-
lates in time. Draw a graph of the $\psi$ versus $t$ showing how a point on the rope at $x = 0$ oscillates. (c) What is the frequency of the wave?

Figure P.2.14

2.15* A transverse wave on a string travels in the negative $y$-direction at a speed of 40.0 cm/s. Figure P.2.15 is a graph of $\psi$ versus $t$ showing how a point on the rope at $y = 0$ oscillates. (a) Determine the wave’s period. (b) What is the frequency of the wave? (c) What is the wavelength of the wave? (d) Sketch the profile of the wave ($\psi$ versus $y$).

Figure P.2.15

2.16 Given the wavefunctions

$$\psi_1 = 4 \sin 2\pi (0.2x - 3t)$$

and

$$\psi_2 = \frac{\sin (7x + 3.5t)}{2.5}$$

determine in each case the values of (a) frequency, (b) wavelength, (c) period, (d) amplitude, (e) phase velocity, and (f) direction of motion. Time is in seconds and $x$ is in meters.

2.17* The wavefunction of a transverse wave on a string is

$$\psi(x, t) = (30.0 \text{ cm}) \cos [(6.28 \text{ rad/m})x - (20.0 \text{ rad/s})t]$$

Compute the (a) frequency, (b) wavelength, (c) period, (d) amplitude, (e) phase velocity, and (f) direction of motion.

2.18* Show that

$$\psi(x, t) = A \sin k(x - vt)$$

is a solution of the differential wave equation.

2.19* Show that

$$\psi(x, t) = A \cos (kx + \omega t)$$

is a solution of the differential wave equation.

2.20* Prove that

$$\psi(x, t) = A \cos (kx + \omega t - \pi/2)$$

is equivalent to

$$\psi(x, t) = A \sin (kx - \omega t)$$

2.21 Show that if the displacement of the string in Fig. 2.7 is given by

$$y(x, t) = A \sin [k(x + \omega t + \epsilon)]$$

then the hand generating the wave must be moving vertically in simple harmonic motion.

2.22 Write the expression for the wavefunction of a harmonic wave of amplitude $10^3 \text{ V/m}$, period $2.2 \times 10^{-13} \text{ s}$, and speed $3 \times 10^8 \text{ m/s}$. The wave is propagating in the negative $x$-direction and has a value of $10^3 \text{ V/m}$ at $t = 0$ and $x = 0$.

2.23 Consider the pulse described in terms of its displacement at $t = 0$ by

$$y(x, t)|_{t=0} = \frac{C}{2 + x^2}$$

where $C$ is a constant. Draw the wave profile. Write an expression for the wave, having a speed $v$ in the negative $x$-direction, as a function of time $t$. If $v = 1 \text{ m/s}$, sketch the profile at $t = 2 \text{ s}$.

2.24* Please determine the magnitude of the wavefunction $\psi(z, t) = A \cos [k(z + vt) + \pi]$ at the point $z = 0$, when $t = \pi/2$ and when $t = 3\pi/4$. 
2.25 Does the following function, in which A is a constant, 
\[ \psi(y, t) = (y - vt)A \]
represent a wave? Explain your reasoning.

2.26* Use Eq. (2.33) to calculate the speed of the wave whose representation in SI units is 
\[ \psi(y, t) = A \cos \pi(3 \times 10^4 y + 9 \times 10^4 t) \]
2.27 Beginning with the following theorem: If \( z = f(x, y) \) and \( x = g(t), y = h(t) \), then 
\[ \frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} \]
Derive Eq. (2.34).

2.28 Using the results of the previous problem, show that for a harmonic wave with a phase \( \phi(x, t) = k(x - vt) \) we can determine the speed by setting \( \frac{dz}{dt} = 0 \). Apply the technique to Problem 2.26 to find the speed of that wave.

2.29* A Gaussian wave has the form \( \psi(x, t) = Ae^{-a^2(x^2 + ct)^2} \). Use the fact that \( \psi(x, t) = f(x - vt) \) to determine its speed and then verify your answer using Eq. (2.34).

2.30 Create an expression for the profile of a harmonic wave traveling in the z-direction whose magnitude at \( z = -\lambda/12 \) is 0.866, at \( z = +\lambda/6 \) is 1/2, and at \( z = \lambda/4 \) is 0.

2.31 Which of the following expressions correspond to traveling waves? For each of those, what is the speed of the wave? The quantities \( a, b, \) and \( c \) are positive constants.

(a) \( \psi(z, t) = (az - bt)^2 \)
(b) \( \psi(x, t) = (ax + bt + cy)^2 \)
(c) \( \psi(x, t) = 1/(ax^2 + b) \)

2.32* Determine which of the following describe traveling waves:

(a) \( \psi(y, t) = e^{-a(y^2 + bt^2 - 2aby)} \)
(b) \( \psi(z, t) = A \sin (ax^2 - bt^2) \)
(c) \( \psi(x, t) = A \sin 2\pi \left( \frac{x}{a} + \frac{t}{b} \right) \)
(d) \( \psi(x, t) = A \cos \frac{1}{2b} x + \cos \frac{1}{2a} t \)

Where appropriate, draw the profile and find the speed and direction of motion.

2.33 Given the traveling wave \( \psi(x, t) = 5.0 \exp (-ax^2 - bt^2 - 2\sqrt{ab} xt) \), determine its direction of propagation. Calculate a few values of \( \psi \) and make a sketch of the wave at \( t = 0 \), taking \( a = 25 \text{ m}^{-2} \) and \( b = 9.0 \text{ s}^{-2} \). What is the speed of the wave?

2.34* Imagine a sound wave with a frequency of 1.10 kHz propagating with a speed of 330 m/s. Determine the phase difference in radians between any two points on the wave separated by 10.0 cm.

2.35 Consider a lightwave having a phase velocity of \( 3 \times 10^7 \text{ m/s} \) and a frequency of \( 6 \times 10^{14} \text{ Hz} \). What is the shortest distance along the wave between any two points that have a phase difference of 30°? What phase shift occurs at a given point in \( 10^{-6} \text{ s} \), and how many waves have passed by in that time?

2.36 Write an expression for the wave shown in Fig. P.2.36. Find its wavelength, velocity, frequency, and period.

![Figure P.2.36 A harmonic wave.](image)

2.37* Working with exponentials directly, show that the magnitude of \( \psi = Ae^{i\omega t} \) is A. Then rederive the same result using Euler's formula. Prove that \( e^{ia} e^{ib} = e^{i(a+b)} \).

2.38* Show that the imaginary part of a complex number \( z \) is given by \( (z - \bar{z})/2i \).

2.39 Beginning with Eq. (2.51), verify that 
\[ \psi(x, y, z, t) = Ae^{i(k(ax + by + cz - wt))} \]
and that \[ \alpha^2 + \beta^2 + \gamma^2 = 1 \]

Draw a sketch showing all the pertinent quantities.

2.40* Show that Eqs. (2.64) and (2.65), which are plane waves of arbitrary form, satisfy the three-dimensional differential wave equation.

2.41 De Broglie’s hypothesis states that every particle has associated with it a wavelength given by Planck’s constant \( h = 6.6 \times 10^{-34} \text{ J.s} \) divided by the particle’s momentum. Compare the wavelength of a 6.0-kg stone moving at a speed of 1.0 m/s with that of light.

2.42 Write an expression in Cartesian coordinates for a harmonic plane wave of amplitude \( A \) and frequency \( \omega \) propagating in the direction of the vector \( \mathbf{k} \), which in turn lies on a line drawn from the origin to the point \((4, 2, 1)\). *Hint:* First determine \( \mathbf{k} \) and then dot it with \( \mathbf{r} \).

2.43* Write an expression in Cartesian coordinates for a harmonic plane wave of amplitude \( A \) and frequency \( \omega \) propagating in the positive \( x \)-direction.

2.44 Show that \( \psi(\mathbf{k} \cdot \mathbf{r}, t) \) may represent a plane wave where \( \mathbf{k} \) is normal to the wavefront. *Hint:* Let \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) be position vectors drawn to any two points on the plane and show that \( \psi(\mathbf{r}_1, t) = \psi(\mathbf{r}_2, t) \).

2.45* Make up a table with columns headed by values of \( \theta \) running from \(-\pi/2\) to \(2\pi\) in intervals of \( \pi/4 \). In each column place the corresponding value of \( \sin \theta \), and beneath those the values of \( 2 \sin \theta \). Next add these, column by column, to yield the corresponding values of the function \( \sin \theta + 2 \sin \theta \). Plot each of these three functions, noting their relative amplitudes and phases.

2.46* Make up a table with columns headed by values of \( \theta \) running from \(-\pi/2\) to \(2\pi\) in intervals of \( \pi/4 \). In each column place the corresponding value of \( \sin \theta \), and beneath those the values of \( \sin(\theta - \pi/2) \). Next add these, column by column, to yield the corresponding values of the function \( \sin \theta + \sin(\theta - \pi/2) \). Plot each of these three functions, noting their relative amplitudes and phases.

2.47* With the last two problems in mind, draw a plot of the three functions (a) \( \sin \theta \), (b) \( \sin(\theta - 3\pi/4) \), and (c) \( \sin \theta + \sin(\theta - 3\pi/4) \). Compare the amplitude of the combined function (c) in this case with that of the previous problem.

2.48* Make up a table with columns headed by values of \( kx \) running from \(-\lambda/2\) to \(+\lambda\) in intervals of \( \lambda/4 \). In each column place the corresponding values of \( \cos kx \) and beneath that the values of \( \cos(kx + \pi) \). Next plot the three functions \( \cos kx \), \( \cos(kx + \pi) \), and \( \cos kx + \cos(kx + \pi) \).