Physics 880.06: Problem Set 4

Note: please turn these problems into the mailbox of the grader, Wissam Al-Saidi, by 12 noon on Monday, May 6. At present, this problem set consists of a single problem, which is worth 20 points.

1. The BCS equation for the gap function $\Delta_k$ at temperature $T = 0$ is

$$\Delta_k = -\frac{1}{2} \sum_{k'} \frac{\Delta_{k'} V_{k,k'}}{\sqrt{\Delta_{k'}^2 + \xi_{k'}^2}}, \quad (1)$$

where $\xi_k = \epsilon_k - E_F$, and $\epsilon_k$ is the energy of a non-interacting electron of wave vector $k$.

(a). Show that if $V_{k,k'} = -A_k A_{k'}$, where $A_k$ is a specified function of $k$, then the above equation is solved by a gap function of the form $\Delta_k = A_k \Delta$, where $\Delta$ is independent of $k$, and find an equation for $\Delta$.

(b). Now suppose specifically that our superconductor is two-dimensional, and that $A_k$ has the form

$$A_k = A_0 \cos(2\phi) \quad (2)$$

if $|\epsilon_k - E_F| < \hbar \omega_c$, and

$$A_k = 0 \quad (3)$$

otherwise. Here $\hbar \omega_c$ is a cutoff energy, $\epsilon_k = \hbar^2 k^2/(2m)$, and we have written $k = (k \cos \phi, k \sin \phi)$. Obtain an integral expression which determines $\Delta$ in this case, but you need not solve this to obtain $\Delta$ explicitly.

(c). The elementary excitations (“Bogoliubons”) described in class are Fermions which have the dispersion relation $E_k = \sqrt{[\Delta A_k]^2 + \xi_k^2}$. Show that these Bogoliubons are gapless - that is, they have vanishing energies at certain specific values of $k$ (NOT at all values of $k$!). What are those values of $k$?

(d). Let $k_0$ be a node point for a Bogoliubon, and let $\delta k = k - k_0$. Show that, if one chooses the coordinate axes of the vector $\delta k$ suitably, the energies of the Bogoliubons sufficiently near the point $k_0$ have the approximate form

$$E(\delta k) \sim \sqrt{C_1[\delta k_1]^2 + C_2[\delta k_2]^2} \quad (4)$$
where $\delta k_1$ and $\delta k_2$ are the components of $\delta k$ in the new coordinate system, and $C_1$ and $C_2$ are positive constants.

(e). What is the density of states of these Bogoliubons at very low energy?

(f). If we neglect the temperature-dependence of the gap variable $\Delta$, what is the functional form of the temperature-dependence of the specific heat at very low temperatures, for a superconductor with this type of energy gap?

Note: This type of gap function is believed to describe some of the cuprate-based high-temperature superconductors.