1. Consider a free electron with an effective mass tensor with principal values $m_x$, $m_y$, and $m_z$, and principal axes parallel to the $x$, $y$, and $z$, axes. That is, assume that its kinetic energy operator $K$, in the absence of a magnetic field, is

$$K = -\frac{\hbar^2}{2} \sum_{i=1}^{3} \frac{1}{m_i} \frac{\partial^2}{\partial x_i^2}. \quad (1)$$

Now suppose that this electron is placed in a magnetic field $B = B\hat{z}$. Calculate the eigenvalue spectrum and calculate the degeneracy of each Landau level. Assume that the system is a parallelopiped of edges $L_x$, $L_y$, and $L_z$, as done in class for the isotropic case.

2. In a system consisting of two spin-1/2 electrons, consider the state $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. Show that this state is an eigenstate of the operator $S^2$ with eigenvalue $S(S+1) = 0$, and of the operator $S_z$ with eigenvalue 0, where $S^2$ is the operator representing the square of the total spin, and $S_z$ is the $z$ component of total spin.

3. This problem is based on the notes handed out in class.

Assuming that the creation and annihilation operators satisfy the anticommutation relations in these notes, and that the states $u_i(\vec{x})$ satisfy the completeness relation

$$\sum_i u_i^\dagger(\vec{x})u_i(\vec{x}') = \delta(\vec{x} - \vec{x}') \quad (2)$$

show that

$$\{\psi(\vec{x}), \psi(\vec{x}')\} = 0 \quad (3)$$
$$\{\psi^\dagger(\vec{x}), \psi^\dagger(\vec{x}')\} = 0 \quad (4)$$
$$\{\psi^\dagger(\vec{x}), \psi(\vec{x}')\} = \delta(\vec{x} - \vec{x}'). \quad (5)$$
(Note that the delta function here is really a product of a Dirac delta function over space coordinates and a Kronecker delta function over spin coordinates.)

Note: Problem set is due by 5PM Friday in either the mailbox of the grader, Sung Yong Park (preferred) or my mailbox.