Periodic Particle Clustering in Narrow Channel Flow

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Abstract: This project involved a preliminary investigation of the periodic clustering of particles in narrow channels under thin film flow. Motivated by particle clustering in runoff after a storm, we constructed an experiment to observe particle clustering behavior. The particle size is comparable to channel width. The channel can be varied in width and angle of inclination. Flow rate can also be varied. Preliminary results indicate that clustering wavelength is proportional to Inclination Angle, and that clustering only occurs for particle densities above a critical density. Detailed results require further investigation.

Motivation

Periodic behaviors in nature have long inspired scientists to seek for greater organizing principles underlying natural phenomena. This fundamental impetus in science is the driving force behind this research project. My mentor, Dr. C. David Andereck, observed periodic clustering of debris in the narrow crease between concrete
Figures 1, 2 and 3 - Periodic Clustering in Crease between Concrete Slabs

slabs along a driveway after a rainfall. He also observed that periodicity varied in different channels (See Figures 1, 2, and 3), suggesting that periodicity depends upon channel width, angle of inclination, flow rate, particle size, particle geometry and perhaps more subtle parameters such as surface roughness, temperature gradients, etc. Seizing the opportunity to put my temporary help to new use, he set me to work exploring this phenomenon.
Purpose

The purpose of my work has been two-fold, to gain an understanding of periodic particle clustering in narrow channel flow and to experience the research environment and research process in its full glory. The second objective naturally followed in pursuit of the first. In order to understand and examine periodic particle clustering in narrow channel flow there were two courses of action to follow. First, to delve into prior research and explore current knowledge on such behavior, and second, to construct an experiment with which to observe, record, and interpret such behavior.

Prior Research

I first explored the wealth of information surrounding laminar flow down an inclined plane. I hoped that this research would shed light on how such simple flows might help explain similar flow with large particulate matter, such as seeds in rainfall runoff. Laminar flow down an inclined plane is a mathematically well formulated problem, first explored by C.S. Yih (1963) and T.B. Benjamin (1957). They analyzed the Orr-Sommerfeld equation with appropriate boundary conditions, found by linearizing the Navier-Stokes equations assuming two-dimensional infinitesimal disturbances (Gollub et al. 1993). This problem has since been extensively explored, and research continues today by a number of scientists around the world. However, two dimensional infinitesimal disturbances on the surface of thin film flow clearly does not describe the behavior of particles clustering in narrow channels where the particles are comparable in size to both fluid height and channel width. Flows with particles in suspension have also been studied at length by U. Schaflinger, A. Acrivos and many others. Like the Orr-Sommerfeld equation, flows with particles in suspension do not describe the behavior of seeds and natural debris clustering along a narrow channel where the size of the particles are comparable to channel width. Ripple formation in layers of sand on riverbeds and in experiment is also well-known. But the periodic formation of particle clusters as shown in Figures 1-3 does not appear to be the result of vortex formation such as that known to cause ripples in sandbeds. Ultimately, I came to the conclusion that very little theoretical or experimental research on periodic particle clustering in narrow channel flow where
particles are comparable to fluid height exists. A paper by Tokunaga, Veerapaneni, and Wan (2000) did describe the forces and torques acting on particles comparable to fluid height and greater in film flows as well as the dependence of particle velocity on the ratio of particle diameter to fluid height. But pattern formation such as periodic particle clustering was not explored or explained. Therefore I turned to experiment for any new understanding of the problem.

**Experimental Apparatus**

The main objective in building the experimental apparatus was to recreate a similar but more controlled channel flow like that expected on the surface of a driveway after a rain. To do this I constructed a plexiglass plane with a plexiglass box at the top. The box is constructed with a slot at the bottom releasing flow onto the plane. The slot is filled with an array of straws to prevent vorticity. There is a gate covering the slot to vary the slot height. The volume of the box is much larger than the outlet to prevent pressure variations. It also contains a pressure release valve to control air pressure. This box is fed by a reservoir full of water maintained at a constant water height. Water recirculates from a reservoir at the bottom of the channel to the upper reservoir by use of a pump, and constant height is maintained using an overflow line from the upper reservoir to the lower reservoir. The adjustable channel is formed by two angled plexiglass pieces clamped onto the plexiglass plane. I also added a layer of mosquito netting stretched over the plane to roughen up the surface and mimic a driveway. Lentils were used as test particles with an average diameter of 7 mm and an average thickness of 3 mm. The experimental schematic is shown in Figure 4.

**Experimental Procedure**

The primary goal of the experiment was to recreate periodic clustering behavior similar to that in Figures 1-3. To do this in a controlled manner I followed a simple procedure. Maintaining a constant water level, I set the plane to a desired angle of inclination and the channel to a desired width, distributed the particles in an even layer
along the plane to a desired density (particles per cm), and then slowly increased the flow rate down the plane. In order to determine how clustering behavior depended upon channel width ($\delta$), inclination angle ($\beta$), and initial particle density ($\rho$, Particles/cm) this procedure was repeated systematically changing only one parameter at a time. Preliminary results are explained in the following section.

Results

The most striking result of the experiment was that narrow channel flow over an initially uniformly distributed layer of particles does result in periodic particle clustering as shown in Figures 6 and 7. The mechanism behind this clustering remains unknown due to a lack of theory, lack of prior research, and the limited time available for the current
Figure 6 – Periodic Particle Clustering, Direction of Flow

Figure 7 – Periodic Particle Clustering (Sideview)
Figure 8 – Clustering Spacing (Wavelength) versus Inclination Angle project. Typically, the clusters would form in the following manner. Increasing the flow rate gradually, particles near the top of the plane would get lifted and flow a small distance down the plane. These particles would push against particles further along the plane until a group of particles became large enough and stable enough to withstand the flow of water. At this point water would build up behind the cluster and overflow, lifting and pushing particles beyond the cluster in the same way. These particles would then start to form a new cluster. This would continue until periodic structures like that in Figures 6 and 7 developed. This behavior was seen to vary for different angles of inclination of the plane. By performing a series of tests at different angles while maintaining channel width, seed type, and seed density I obtained a variation in “wavelength” (cluster spacing) with angle as shown above in blue. The least squares linear fit to this data is shown in pink. The most definitive conclusion to be drawn from this data is that wavelength $\lambda$ is directly proportional to inclination angle $\beta$.

$$\lambda \sim \beta$$

However, more detailed experiments as to how flow rate affects cluster spacing at different angles are required to determine the importance of inclination angle alone. I
had only the ability to perform crude flow rate measurements, but experiment suggests that once stable clusters form flow rate does not alter wavelength at a given inclination angle. However, all of the experiments were done increasing flow rate gradually, more drastic changes in flow may cause different behaviors, and require better flow rate measurement techniques.

I also found that particle clustering will only occur if the ratio of particle density along the channel \( \rho \), particles per cm, to channel width \( \delta \) (i.e. particles/cm\(^2\)) is greater than a critical density \( \sigma \).

Clustering occurs if: \[ \rho / \delta > \sigma \]

This result came from a series of tests in which channel width was gradually increased maintaining constant seed density until particle clustering no longer occurred. This experiment was repeated for a variety of seed densities at an inclination angle around 8 degrees. At this angle, for each seed density particle clustering stopped when the ratio of seed density to channel width was around 0.85 particles/cm\(^2\). This suggests that this ratio is a critical density below which clustering will not occur. How critical density might vary with inclination angle remains to be seen. The data also seemed to suggest that wavelength (cluster spacing), increased as channel width increased so long as the particle density to channel width ratio remained the same.

While preliminary results suggest a simple relationship between wavelength and inclination angle as well as the existence of a critical density below which periodic clustering will not occur, further experimentation is required to fully understand the particle clustering problem. It is unclear how wavelength depends upon channel width, flow rate and seed density. It is also unclear how parameters such as surface roughness and particle type affect wavelength. The underlying functional dependence of periodic particle clustering wavelength on all of these parameters is completely unknown, as is the mechanism behind the clustering. A full understanding of periodic particle clustering in narrow channel flow will require a wealth of statistics to formulate conclusively.
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References