For a single bullet to reverse course, with its velocity changing from +500 m/s to -500 m/s, requires a change in momentum of

\[ \Delta p = p_f - p_i = mv_f - mv_i = (0.03 kg)(-500 - 500) = -3.0 kg \cdot m/s \]

In a single minute, this happens 100 times, so

\[ \Delta p_{\text{tot}} = -300 \text{ kg} \cdot \text{m/s} \]

And since Superman’s chest is providing all the force needed to exert this impulse in the space of 60 seconds, he feels a force of

\[ F = \frac{-\Delta p_{\text{tot}}}{\Delta t} = \frac{300}{60} = 5 \text{ N} \]

This is a pretty pony force. (But it is correct.) Why can’t we Earthlings shrug off machine-gun attacks if they exert so little force when averaged over a whole minute-long barrage?

We know that impulse is \( \vec{J} = \Delta \vec{p} = \int_{t_1}^{t_2} F(t) \, dt \).

So, the total change in momentum is the area under the graph. From the given info, we can find that

\[ \vec{J} = \vec{p}_f - \vec{p}_i = (0.058 kg)(34 m/s) - (0.058 kg)(-34 m/s) \]

\[ = 3.9 \text{ kg} \cdot \text{m/s} \]

This has to equal the area under the \( F(t) \) graph:

\[
\text{Area} = A_{\text{I}} + A_{\text{II}} + A_{\text{III}} = \frac{1}{2}(2s)F_{\text{max}} + (2s)F_{\text{max}} + \frac{1}{2}(2s)F_{\text{max}} = 2(2s)F_{\text{max}} = (4s)F_{\text{max}}
\]

So...

\[
\text{Area} = \vec{J} \rightarrow (4s)F_{\text{max}} = 3.9 \text{ kg} \cdot \text{m/s}
\]

\[
F_{\text{max}} = 0.97 \text{ N}
\]
\( F(t) = \left[ (6.0 \times 10^6 t - (2.0 \times 10^9) t^2 \right] N \)

for \( 0 \leq t \leq 3.0 \times 10^{-3} \) s

(a) Find the impulse on the ball from \( t=0 \) to \( t = 3.0 \times 10^{-3} \) s

\[
\overline{J} = \int_{t_1}^{t_2} F(t) \, dt \\
= \int_{t_1}^{t_2} \left[ (6.0 \times 10^6 t - (2.0 \times 10^9) t^2 \right] dt \\
= \left[ \frac{3.0 \times 10^6 t^2}{2} - \frac{2.0 \times 10^9 t^3}{3} \right]_{t_1=0}^{t_2=3.0 \times 10^{-3}} \]

\[
= 27 \quad - \quad 18 \\
= \boxed{9 \, N \cdot \text{s} = J}
\]

(b) Find \( F_{avg} \)

\[
F_{avg} = \frac{\overline{J}}{t_2 - t_1} = \frac{9 \, N \cdot \text{s}}{(3.0 \times 10^{-3}) \text{s}}
\]

\[
F_{avg} = \boxed{3000 \, \text{N}}
\]

(c) Find maximum force, let's take a derivative to find \( t \) of max force
\[
\frac{dF(t)}{dt} = 0 = 6.0 \times 10^6 - 4.0 \times 10^9 t
\]

\[\Rightarrow t = \frac{6.0 \times 10^6}{4.0 \times 10^9} = 1.5 \times 10^{-3}\]

Let's make sure this is a max, take second derivative

\[\frac{d^2F(t)}{dt^2} = -4.0 \times 10^9 < 0 \Rightarrow \text{max}\]

\[F_{\text{max}} = F(1.5 \times 10^{-3}, \text{cc}) = (6.0 \times 10^6)(1.5 \times 10^{-3}) - (2.0 \times 10^9)(1.5 \times 10^{-3})^2 \]

\[= \boxed{14500 \text{ N } = F_{\text{max}}}\]

d) The ball's max speed as it loses contact w/ the player's foot,

\[J = \Delta p = p_f - p_i = mv_f - mv_i, \quad v_i = 0\]

\[\Rightarrow v_f = \frac{J}{m} = \frac{9 \text{ m/s}}{1.5 \text{ kg}} = \boxed{20 \text{ m/s } = v_f}\]
Before the bullet hit the block

After the bullet emerges.

The momentum of the bullet-block system

\[ P_i = 5.2 \times 10^{-3} \times 672 + 700 \times 10^{-3} \times 0 \]

\[ P_f = 5.2 \times 10^{-3} \times 428 + 700 \times 10^{-3} \times v_f \]

No matter what kind of collisions it is, momentum must be conserved.

\[ P_i = P_f \]

\[ 5.2 \times 10^{-3} \times 672 = 5.2 \times 10^{-3} \times 428 + 700 \times 10^{-3} \times v_f \]

\[ v_f = 1.81257 \text{ (m/s)} \]

The center of mass for the system

\[ v_{cm,i} = \frac{5.2 \times 10^{-3} \times 672 + 700 \times 10^{-3} \times 0}{(5.2 + 700) \times 10^{-3}} \]

\[ = 4.955 \text{ (m/s)} \]

\[ v_{cm,f} = \frac{5.2 \times 10^{-3} \times 428 + 700 \times 10^{-3} \times 1.8126}{(5.2 + 700) \times 10^{-3}} \]

\[ = 4.955 \text{ (m/s)} \]

Actually the center of mass will keep moving with the same speed.

(You might need to use more decimals for \( v_f \) to get the same answers.)

But total energy of the system will not be the same. The system will lose some energy when the bullet emerges.
I-5
10-25 (B)

\( m_A = 1100 \text{ kg} \)
\( m_B = 1400 \text{ kg} \)
\( f_k = 0.13 \)

A stops

\( (\Delta x)_A = 8.2 \text{ m} \)

\( (\Delta x)_B = 6.1 \text{ m} \)

a) \( v_A = ? \) (After Impact)
b) \( v_B = ? \) (Right after Impact)
c) \( v_B \) (before collision) using conservation of momentum

a) The only force on cars is friction since they try to stop.

We need to find out their deceleration from Newton's second law:

\[
F = ma
\]
\[
f_N = ma \quad a = \frac{f_N}{m} = \frac{-f_kN}{m} \quad \rightarrow \quad a = \frac{f_kN}{m}
\]

We have the stopping distance for each car and knowing they will stop:

\[
v^2 - v_0^2 = 2a(\Delta x)
\]

For A, \( v_0 = 0 \) \( (\Delta x)_A = 8.2 \text{ m} \) \( \Rightarrow v_A = \sqrt{2a(\Delta x)_A} \)
\[ V_A = \sqrt{2 \left( \mu_k g \right) (\Delta x)_A} = \sqrt{2 \left( 0.13 \right) (9.8)(8.2)} \]

\[ V_A = 4.6 \text{ m/s} \]

b) \[ V_B = \sqrt{2 \left( \mu_k g \right) (\Delta x)_B} = \sqrt{2 \left( 0.13 \right) (9.8)(6.1)} \]

\[ V_B = 3.9 \text{ m/s} \]

c) Using conservation of linear momentum:

Total momentum before impact = total momentum after impact

\[ m_B v_B' \text{(before Impact)} + 0 = m_A v_A \text{(After)} + m_B v_B \text{(After)} \]

\[ v_B' = \frac{m_A v_A + m_B v_B}{m_B} = \frac{(100)(4.6) + (1400)(3.9)}{1400} \]

\[ v_B' = 7.5 \text{ m/s} \]

We can use the conservation of linear momentum only if there is no other force acting on two cars beside (contact between two cars) during the Impact. This means during Impact (Δt) we neglect the frictional force by the road on the cars. Also in sliding friction force is more complicated than just \( \mu_k \cdot m g \) that we assumed here.
As hinted in the problem statement, when the spring reaches the maximum compression $x_m$, the whole system has velocity $v$ and satisfies (inelastic) momentum conservation:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$$

So $v$ is

$$v = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{2(10) + 5(3)}{2 + 5} = 5 \text{ m/s}$$

Let's look at the change in kinetic energy:

$$\Delta K = K_f - K_i = \left(\frac{1}{2} (m_1 + m_2) v^2\right) - \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2\right)$$

$\uparrow$

$$5 \text{ m/s}$$

Everything else in this form above is known.

$$\Delta K = \frac{1}{2} (2 + 5) 5^2 - \left(\frac{1}{2} (2)(10^2) + \frac{1}{2} (5)(3^2)\right) = -35 \text{ J}$$

This change in kinetic energy goes into the compression of the spring and is all stored in the potential energy of the spring at the particular moment in question (when we have maximum compression).

So $U_{sp} = -\Delta K$, energy conservation

$$\frac{1}{2} k x_m^2 = 35 \text{ J}$$

$$x_m = \sqrt{\frac{2(35)}{k}} = \sqrt{\frac{70}{120} \text{ J}} = 0.25 \text{ m}$$

$$x_m = 25 \text{ cm}$$
The collision is elastic so energy is conserved

A) \( KE_i = KE_f \Rightarrow \frac{1}{2} m_1 v_{i0}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 \)

\[ \frac{1}{2} m_1 v_{i0}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 \quad \text{or} \quad \frac{1}{2} m_2 v_{f2}^2 = 1.70 J \]

there are 2 unknowns in this equation
so we need another equation.

momentum is always conserved so:

\[ M_i v_{i0} + M_0 v_{i0} = M_1 v_{f1} + M_2 v_{f2} \]

plug this back into first equation to get:

\[ \frac{1}{2} m_2 \frac{m_1^2 (v_{i0} - v_{i2})^2}{m_2} = 1.70 J \]

or \( m_2 = \frac{m_1^2 (v_{i0} - v_{i2})^2}{(2) 1.70 J} = 0.999 \) kg

B) \( V_{f2} = \frac{m_1 (v_{i0} - v_{i2})}{m_2} = 1.85 m/s \)

C) \( V_{cm} = \frac{1}{M} \sum m_i v_i = \frac{1}{m_1 + m_2} \left( m_1 v_1 + m_2 v_2 \right) \)

\[ V_{cm} = 0.93 \) m/s \]
We can treat this as a one dimensional collision...

By (10.38)

\[ V_{\text{Voyage}} = \frac{m v - M_J}{m v + M_J} V_{\text{voy},0} + \frac{2 M_S}{m v + M_J} V_{\text{Sup},0} \]

since \( m v \ll M_J \) we can completely ignore \( m v \)...

\[ V_{\text{Voyage}} = -\frac{M_J}{M_J} V_{\text{voy},0} + \frac{2 M_S}{M_J} V_{\text{Sup},0} \]

\[ V_{\text{voy}} = V_{\text{Voyage},0} + 2 V_{\text{Sup},0} \quad !!! \quad \text{a huge gain...} \]

\[ = -12 \text{ km/s} \quad + 2(-13 \text{ km/s}) \]

\[ V_{\text{voy}} = -38 \text{ km/s} \]

\[ \frac{|V_f - V_i|}{|V_i|} \times 100\% = 217\% \text{ gain!} \]

But what is the cost of this increase in speed? i.e. what is \% change in \( V_{\text{Sup}} \)?

(10.39) \[ V_{\text{Sup}} = \frac{2 m v}{m v + M_J} V_{\text{voy},0} + \frac{M_J - m v}{M_J + m v} V_{\text{Sup}} \]

Again we can write \( M_J + m v \approx M_J \)

\[ V_{\text{Sup}} = 2 \frac{m v}{M_J} V_{\text{voy},0} + V_{\text{Sup}} \]

\[ \frac{|V_f - V_i|}{|V_i|} \times 100\% = \frac{2 m v}{M_J} \frac{12 \text{ km/s}}{13 \text{ km/s}} \times 100\% \text{ Let } M_J = 1.9 \times 10^{27} \text{ kg} \]

\[ \text{mv} = 800 \text{ kg} \]

\[ = \frac{(800 \text{ kg})}{(1.9 \times 10^{27} \text{ kg})} (1.846) 100\% \]

\[ \frac{|V_f - V_i|}{|V_i|} = 7.9 \times 10^{-23}\% \text{ loss !!!} \]
We’ll start this solution with a derivation of equations (10-30) and (10-31). If you don’t care, and just want to use those equations, go to the next page.

In the first collision, conserving momentum and K.E., we get

\[ m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]
\[ \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \]
\[ m_2 (v_{2i} - v_{2f}) = m_1, V_{1f} \]
\[ \frac{1}{2} m_2 (v_{2i}^2 - v_{2f}^2) = \frac{1}{2} m_1, v_{1f}^2 \]
\[ m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f}) = m_1, v_{1f}^2 \]

Divide the energy equation by the momentum equation:

\[ \frac{m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f})}{m_2 (v_{2i} - v_{2f})} = \frac{m_1, v_{1f}^2}{m_1, v_{1f}} \rightarrow v_{1f} = v_{2i} + v_{2f} \]

Plugging this result back into the energy equation gives us

\[ m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f}) = m_1, (v_{2i} + v_{2f})^2 \]
\[ m_2 (v_{2i} - v_{2f}) = m_1, (v_{2i} + v_{2f}) \]
\[ m_2 v_{2i} - m_2 v_{2f} = m_1 v_{2i} + m_1 v_{2f} \]
\[ V_{2f} = V_{2i} \left( \frac{m_2 - m_1}{m_2 + m_1} \right) \]

Using the division result again,

\[ v_{1f} = v_{2i} + v_{2f} = v_{2i} + v_{2i} \left( \frac{m_2 - m_1}{m_2 + m_1} \right) \]
\[ = v_{2i} \left( \frac{m_2 + m_1}{m_2 + m_1} \right) + v_{2i} \left( \frac{m_2 - m_1}{m_2 + m_1} \right) \]
\[ v_{1f} = v_{2i} \left( \frac{2 m_2}{m_2 + m_1} \right) \]

Now let’s turn our attention to the next collision...
One thing to note is that our problem has $m_1$ initially stationary, but
the book derives (10-30) and (10-31) with $m_2$ stationary, so all the
subscripts are reversed between ours and theirs.

In the second collision, when $m_2$ hits the wall, its velocity simply reverses
without changing its magnitude. So after hitting the wall, the velocity
of $m_2$ changes from $v_{2f}$ to $-v_{2f}$. For the problem, we want that
velocity, $-v_{2f}$, to equal $v_{1f}$. So, using the results from the last page...

$$v_{1f} = -v_{2f}$$

$$v_{2i} \left( \frac{2m_2}{m_1 + m_2} \right) = v_{2i} \left( \frac{m_1 - m_2}{m_1 + m_2} \right)$$

$$2m_2 = m_1 - m_2$$

$$3m_2 = m_1 \rightarrow m_2 = \frac{m_1}{3}$$
For 2-D collisions, you need to find the x and y components of the momenta and do a conservation statement for both directions. Make sure that any energy conservation statement uses the total velocity, and not a single component.

There is no such thing as $K_x$ and $K_y$, for example.

(a) I've arranged my coordinate axes so that the cue's original velocity was in the $+x$ direction. I don't know what that velocity was, so I can't use $x$-conservation in the $x$-direction. I do know that both balls had initial $y$-velocity of zero, though, so $p_y = 0$. Using that...

\[ p_y = pf \]
\[ 0 = m_{cue}v_{ey} + m_{by}v_{ey} = m(3.5 \sin 22^\circ) - m(2.0 \sin \theta) \]

I've assigned a minus sign to $v_{by}$ in the last step, since I can tell it belongs there from my diagram. So, solving for $\theta$, I find

\[ \theta = \arcsin \left( \frac{3.5}{2.0} \sin [22^\circ] \right) = 41^\circ \]

(b) I don't know for sure, yet, if the collision is elastic, so to find the cue's initial speed I'll use momentum instead of energy. Let's look in $x$-direction:

\[ p_x = pf \]
\[ m_{cue} = m_{cue,xf} + m_{by,xf} \]
\[ = m(3.5 \cos 22^\circ) + m(2.0 \cos 41^\circ) \rightarrow v_{cue,i} = \boxed{4.75 \text{ m/s}} \]

(c) Now we know all the corresponding velocities and can check the energy:

\[ \frac{1}{2}m_{cue,i}^2 \geq \frac{1}{2}m_{cue,xf}^2 + \frac{1}{2}m_{by,xf}^2 \]
\[ \frac{1}{2}(4.75)^2 \geq \frac{1}{2}(3.5)^2 + \frac{1}{2}(2.0)^2 \]

\[ 22.6 \geq 16.2 \]

Nope! Energy was lost in the collision!