After the ship separates, one part will be going faster than the initial velocity, and one will be slower than the initial velocity. (Or, maybe, even have a negative v.) Graphs (a), (d), and (f) show this characteristic.

b. The trailing component must have a lower velocity, so that's line #2.

c. The angle between line 1 and 2 is a measure of the relative speeds, since the slopes are the velocities. So the greatest relative speed is in (d), then (f), then (a).

2. \( M_\oplus = 5.98 \times 10^2 \) kg
\( M_D = 7.36 \times 10^2 \) kg
\( d_{\oplus-D} = 3.82 \times 10^8 \) m

(a) Putting the origin at Earth simplifies the math...
\[ x_{com} = \frac{M_\oplus x_\oplus + M_D x_D}{M_\oplus + M_D} = \frac{M_D d_{\oplus-D}}{M_\oplus + M_D} = 4.64 \times 10^6 \text{ m} \]

(b) \( R_\oplus = 6400 \text{ km, so} \)
\[ \frac{x_{com}}{R_\oplus} = \frac{4.64 \times 10^6 \text{ m}}{6400 \times 10^3 \text{ m}} \rightarrow x_{com} = 0.726 R_\oplus \]
\( l_{He} = 9.4 \times 10^{-11} \text{ m} \)

\( M_N = 13.9 \text{ m}_H, \ l_{N-H} = 10.14 \times 10^{-11} \text{ m} \)

Set the \( z \)-axis pass \( N \) & the center of mass of the 3 Hs.

At \( N, \ \theta = 0 \),

At C.M. of Hs, \( \delta H = \sqrt{l_{NH}^2 - l_{HC}^2} \)

\[ = 3.8 \times 10^{-11} \text{ m} \]

The center of mass for \( NH_3 \) will be at

\[ \delta = \frac{m_N \cdot 0 + 3m_H \cdot \delta_H}{m_N + 3m_H} = \frac{-3 \times 3.8 \times 10^{-11} m_H}{(13.9 + 3) m_H} \]

\[ = -6.746 \times 10^{-12} \text{ m} \]
a.) If the man climbs with a speed $V$, what speed and in what direction will the balloon move relative to the ground?

System: First, define the system as the balloon, basket, ladder and man. The force body diagram is:

where $F_B$ is the force of the hot air.

Since the system is initially at rest, \[ F_B = -F_g \quad \text{and} \quad F_{net} = 0. \]

$F_{net}$ acts on the center of mass, and as the man begins to climb, the net force of the system remains $F_{net} = 0$.

\[ x_{com}(t) = v_0 t + \frac{1}{2} a_{com} t^2 \rightarrow \dot{v} = 0 \]

\[ a_{com} = \frac{F_{net}}{m+m} = 0 \]

\[ x_{com}(t) = x_{com}(0) \quad C\text{onstant.} \]

But within the system, the center of mass will shift relative to the man and balloon to maintain $x_{com} = x(t)$.

So:

\[ x_{com}(t) = \frac{B_{com}(t) M}{M+m} + \frac{H_{com}(t) M}{M+m} = x(0) \]

where: $B_{com}(t)$ is the balloon center of mass

$H_{com}(t)$ is the human center of mass.

So:

\[ \frac{d}{dt} x_{com} = \frac{d}{dt} \frac{B_{com}(t)}{M} + \frac{d}{dt} \frac{H_{com}(t)}{M} = 0 \]

where \[ \frac{d}{dt} \frac{H_{com}(t)}{M} = V \]

Speed of balloon:

\[ \frac{d}{dt} \frac{B_{com}(t)}{M} = -V \frac{M}{M} \quad \text{negative means downward direction} \]

b.) What happens when the man stops climbing?

$V \rightarrow 0$ \[ \frac{d}{dt} \frac{B_{com}(t)}{M} = -V \frac{M}{M} = 0 \quad \text{The balloon stops moving...} \]
This problem presents a real-world situation in which you can take practical advantage of your physics training: secretly calculating your date's mass based on the motion of your shared canoe.

Because there are no external forces on the system, the position of the center of mass must be constant. That is, before and after the switch, $X_{\text{com}}$ must be the same. So, let's set up the problem.

\[
M_A = 80 \text{ kg} \\
M_c = ? \\
M_{\text{boat}} = 30 \text{ kg}
\]

\[
\text{Initial:} \quad \begin{array}{c}
\text{Carmelita} \\
\text{Boat}
\end{array}
\]

In the initial state, we place our origin at the left seat, so Carmelita's position is zero.

\[
X_c = 0, \quad X_{\text{boat}} = 1.5 \text{ m}, \quad X_2 = 3.0 \text{ m}
\]

The C.O.M. is then somewhere between 1.5 m and 3.0 m. (Closer to Ricardo.)

Afterwards, the canoe has shifted by .4 m, according to the problem. So the left seat, now occupied by Ricardo, is at .4:

\[
X_c' = .4 \text{ m}, \quad X_2 = .4 + 1.5 \text{ m}, \quad X_c' = .4 + 3 \text{ m}
\]

Sticking these data into the C.O.M. formula:

\[
X_{\text{com}} = X_{\text{com}}'
\]

\[
\frac{M_c X_c + M_{X_R} + M_B X_B}{M_c + M_{X_R} + M_B} = \frac{M_c X_c' + M_{X_R}' + M_B X_B'}{M_c + M_{X_R} + M_B}
\]

The denominators cancel out, since they are identical. Also, $X_c = 0$, so,

\[
M_c X_c = M_c X_c' + M_{X_R}' + M_B X_B'
\]

\[
80 \cdot 3 + 30 \cdot 1.5 = M_c \cdot 3.4 + 80 \cdot .4 + 30 \cdot 1.9
\]

\[
M_c = \frac{80 \cdot 3 + 30 \cdot 1.5 - 80 \cdot .4 - 30 \cdot 1.9}{3.4}
\]

\[
M_c = 57.6 \text{ kg}
\]

(Around 130 pounds.)
\[ v_1 = \frac{52 \text{ km/h}}{1} \]

\[ \frac{m_1 v_1}{2} = \frac{m_2 v_2}{2} \]

\[ (816 \text{ kg}) (v_1) = (2650 \text{ kg}) (16 \text{ km/h}) \]

\[ v_1 = 52 \text{ km/h} \]

\[ \sqrt{\frac{m_2}{m_1}} = \frac{v_2}{v_1} \]

\[ v_1 = 29 \text{ km/h} \]
This problem mixes momentum conservation with some work/energy ideas.

\[ V_0 = 7600 \text{ m/s} \quad m_{\text{rocket}} = 290 \text{ kg} \]
\[ V_{\text{rel}} = 910 \text{ m/s} \quad m_{\text{payload}} = 150 \text{ kg} \]

(a) Initially, both components of the rocket are moving at \( V_0 \). Let's assume the "payload" section gets pushed forward while the (spent) case lags behind. So,

\[ p_i = (m_{\text{rocket}} + m_{\text{payload}}) V_0 \]
\[ p_f = m_{\text{rocket}} V_{\text{rocket}} + m_{\text{payload}} V_{\text{payload}} \]

And with the given relative velocity, \( V_{\text{rocket}} = V_{\text{rocket}} + V_{\text{rel}} \), and \( p_i = p_f \), we write

\[
\begin{align*}
(m_{\text{rocket}} + m_{\text{payload}}) V_0 &= m_{\text{rocket}} V_{\text{rocket}} + m_{\text{payload}} (V_{\text{rocket}} + V_{\text{rel}}) \\
&= V_{\text{rocket}} (m_{\text{rocket}} + m_{\text{payload}}) + m_{\text{payload}} V_{\text{rel}} \\
\rightarrow V_{\text{rocket}} &= \frac{(m_{\text{rocket}} + m_{\text{payload}}) V_0 - m_{\text{payload}} V_{\text{rel}}}{m_{\text{rocket}} + m_{\text{payload}}} = \frac{(290+150)7600 - (150)910}{290 + 150} = 7290 \text{ m/s}
\end{align*}
\]

(b) \( V_{\text{payload}} = V_{\text{rocket}} + V_{\text{rel}} = 7290 + 910 = 8200 \text{ m/s} \)

(c) \( K_i = \frac{1}{2} m_{\text{rocket}} V_0^2 + \frac{1}{2} m_{\text{payload}} V_0^2 \]
\[ = \frac{1}{2} (290)(7600)^2 + \frac{1}{2} (150)(7600)^2 = \frac{1.2707 \times 10^9}{10} \text{ J} \]

(d) \( K_f = \frac{1}{2} m_{\text{rocket}} V_r^2 + \frac{1}{2} m_{\text{payload}} V_p^2 \]
\[ = \frac{1}{2} (290)(7290)^2 + \frac{1}{2} (150)(8200)^2 = \frac{1.2749 \times 10^9}{10} \text{ J} \]

Note, there's a difference in the energies at the fourth digit. (Normally, I wouldn't write so many digits, but I wanted to see the difference.) The final kinetic energy is slightly higher because energy stored in the spring was transformed to kinetic energy.
(9-38) \quad \rightarrow \quad M_0 v_0

So \quad KE_0 = \frac{1}{2} M_0 v_0^2

then after the collision:

\[ \begin{align*}
  & M_1, v_1 \quad \rightarrow \quad M_2, v_2 \\
  & M_1 = \frac{1}{4} M_0 \\
  & M_2 = \frac{3}{4} M_0 \\
  & v_1 = 0 \\
  & v_2 = ?
\end{align*} \]

Conservation of momentum says:

\[
P_0 = P_f \quad \Rightarrow \quad M_0 v_0 = M_2 v_2 = \frac{3}{4} M_0 v_2
\]

\[
  \Rightarrow \quad v_2 = \frac{4}{3} v_0
\]

\[
  KE_f = \frac{1}{2} M_2 v_2^2 = \frac{1}{2} (\frac{3}{4} M_0) \left( \frac{4}{3} v_0 \right)^2
\]

\[
  KE_f = \left( \frac{1}{5} \right) \left( \frac{4}{3} \right) M_0 v_0^2 = \left( \frac{2}{3} \right) M_0 v_0^2
\]

Now \quad \Delta KE = KE_f - KE_i

\[
  \Delta KE = \frac{3}{2} M_0 v_0^2 - \frac{1}{2} M_0 v_0^2
\]

\[
  = \frac{4}{6} M_0 v_0^2 - \frac{3}{6} M_0 v_0^2
\]

\[
  \Delta KE = \frac{1}{6} M_0 v_0^2
\]
During a lunar mission, it is necessary to increase the speed of a spacecraft by 2.2 m/s when it is moving at 400 m/s relative to the Moon. The speed of the exhaust products from the rocket engine is 1000 m/s relative to spacecraft. What fraction of the initial mass of the spacecraft must be burned and ejected to accomplish the increase?

The velocity with respect to the moon is not relevant.

If we are sitting on the spacecraft we see it as moving with zero velocity.

Then if we have

\[ V_e = 1000 \text{ m/s} \quad V_R = 2.2 \text{ m/s} \]

we have to conserve momentum

\[ m_e V_e = (m_R - m_e) V_R \]

\[ m_R = \text{mass of Rocket} \]

\[ m_e = \text{mass ejected} \]

\[ \Rightarrow \text{we want} \quad \frac{m_e}{m_R} \quad \frac{V_e}{V_R} = \frac{m_R - m_e}{m_e} = \frac{m_R}{m_e} - 1 \]

\[ 1 + \frac{V_e}{V_R} = \frac{m_R}{m_e} \Rightarrow \frac{m_e}{m_R} = \frac{1}{\frac{V_e}{V_R} + 1} \]

\[ \frac{m_e}{m_R} = \frac{1}{\frac{(1000)}{(2.2)} + 1} = \frac{2.2 \times 10^{-3}}{1} = \frac{m_e}{m_R} \]
\[ a) \]

\[ \Delta U = mgh = \Delta E_{\text{internal}} \]

\[ = 90(9.8)(8850) = 7.8 \times 10^6 \text{ J} \]

\[ b) \quad 7.8 \times 10^6 \text{ J} \times \frac{1 \text{ bar}}{1.25 \times 10^6 \text{ J}} \approx 6 \text{ bars} \]