Problem 1 (20 points) A mass $M$ can slide without friction on a horizontal surface in the $x$ direction. A mass $m$ is suspended from $M$ by a massless rod of length $L$, and can swing freely in the vertical plane.

(a) Find the Lagrangian. (10 points)

(b) Is there a cyclic coordinate? If so find the corresponding conserved momentum. (5 points)

(c) Find the Lagrange equations of motion (you do not have to solve these equations). (5 points)
Problem 2 (20 points) Three point masses, each of mass \( m \), can slide on a circle of radius \( R \). They are connected by springs of spring constant \( k \). When the masses are evenly spaced around the circles, the springs are at their relaxed lengths.

![Diagram of three masses connected by springs on a circle](image)

(a) Use the angular positions \( \theta_1, \theta_2, \theta_3 \) as variables. Find the frequencies of small oscillations and the normal modes. (15 points)

(b) Suppose at \( t = 0 \) the displacements were (\( \epsilon \) is a small number)

\[
\theta_1 = \epsilon, \quad \theta_2 = 0, \quad \theta_3 = 0
\]

(1)

and \( \dot{\theta}_1 = 0, \dot{\theta}_2 = 0, \dot{\theta}_3 = 0 \). Find the evolution of the oscillations for later times. (5 points)