Physics 5300, Theoretical Mechanics  Spring 2015

Quiz 8

Given: Friday March 6

Problem 1  (a) Find the Poisson bracket of \( f = p^2 q + q^2 \) and \( g = pq \). (3 points)
(b) Suppose we wish to go from variables \( q, p \) to variables \( Q(q,p), P(q,p) \). We set
\[
Q = \frac{p}{1 - 3q}
\]  
Find a possible choice of \( P \). (7 points)

Solution:  (a) We have
\[
\{ f, g \} = \frac{\partial f}{\partial q} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial q} = (p^2 + 2q)(q) - (2pq)(p) = p^2 q + 2q^2 - 2p^2 q = 2q^2 - p^2 q
\]
(b) We use a generating function \( F(q,Q) \). Then we have
\[
p = \frac{\partial F}{\partial q}
\] \[ \quad \]
\[
P = -\frac{\partial F}{\partial Q}
\] \[
We have
\[
p = Q(1 - 3q)
\]
The first relation gives
\[
\frac{\partial F}{\partial q} = Q(1 - 3q)
\]
This integrates to
\[
F = Q(q - \frac{3q^2}{2}) + g(Q)
\]
We set \( g(Q) = 0 \). This gives
\[
F(q,Q) = Q(q - \frac{3q^2}{2})
\]
This then gives
\[
P = -\frac{\partial F}{\partial Q} = -\left[q - \frac{3q^2}{2}\right] = \frac{3q^2}{2} - q
\]
Alternatively, we can guess the form of \( P \), and check:
\[
\{ Q, P \} = 0 - (\frac{1}{1 - 3q})(3q - 1) = 1
\]

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