Problem 1  A particle moves in a circle (center $O$ and radius $R$) with constant angular velocity $\omega$ counterclockwise:

$$\vec{r}(t) = R \cos(\omega t)\hat{x} + R \sin(\omega t)\hat{y}$$

Find

(i) The velocity at time $t$. (2 points)

$$\vec{v}(t) = -R\omega \sin(\omega t)\hat{x} + R\omega \cos(\omega t)\hat{y}$$

(ii) The acceleration at time $t$. (2 points)

$$\vec{a}(t) = -R\omega^2 \cos(\omega t)\hat{x} - R\omega^2 \sin(\omega t)\hat{y}$$

(iii) What is the magnitude and direction of the acceleration? (describe in any way you like, but the result should be clear). (2 points)

$$|\vec{a}|^2 = R\omega^2$$

We see that

$$\vec{a} = -R\omega^2 \hat{x}$$

Thus the acceleration points radially inwards.

Problem 2  The unit vector $\hat{r}$ in 2-d polar coordinates is equal to

$$\hat{r} = \cos \phi \hat{x} + \sin \phi \hat{y}$$

Find the corresponding expression for the unit vector $\hat{\phi}$. (4 points)

$\hat{\phi}$ is perpendicular to $\hat{r}$, and at $\phi = 0$, it points in the positive $y$ direction. Thus writing

$$\hat{\phi} = a \hat{x} + b \hat{y}$$

We see that

$$a \cos \phi + b \sin \phi = 0$$
Thus
\[
\frac{a}{b} = -\frac{\sin \phi}{\cos \phi}
\]
so we can write
\[
a = -k \sin \phi, \quad b = k \cos \phi
\]
for some constant \(k\). We also have, since \(\hat{\phi}\) is a unit vector
\[
a^2 + b^2 = 1
\]
This gives
\[
k = \pm 1
\]
so we have the two possibilities
\[
\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}
\]
and
\[
\hat{\phi} = \sin \phi \hat{x} - \cos \phi \hat{y}
\]
Using the last geometric condition that \(\hat{\phi}\) point in the counterclockwise direction along the circle, we get
\[
\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}
\]