Azimuthal dependence of pion interferometry at the AGS

E895 Collaboration


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Abstract

Two-pion correlation functions, measured as a function of azimuthal emission angle with respect to the reaction plane, provide novel information on the anisotropic shape and orientation of the pion-emitting zone formed in heavy ion collisions. We present the first experimental determination of this information, for semi-central Au + Au collisions at 2–6 A GeV. The source extension perpendicular to the reaction plane is greater than the extension in the plane, and tilt of the pion source in coordinate space is found to be opposite its tilt in momentum space. © 2000 Elsevier Science B.V. All rights reserved.

Nuclear collisions at finite impact parameter are intrinsically anisotropic with respect to the azimuthal angle about the beam direction. For nearly two decades, a growing community has studied the details of the momentum-space anisotropy of emitted particles (directed and elliptic flow) at all collision ener-
gies [1]. Theoretical studies [2–7] suggest that coordinate-space anisotropies are equally interesting. Recently, a formalism was proposed to use azimuthally-sensitive intensity interferometry, correlated event-by-event with the reaction plane, to extract this information experimentally [5,6]. In this Letter, we present the first experimental measurement of the full coordinate-space anisotropies of a hot nuclear source, from pions emitted from Au + Au collisions at 2–6 A GeV.

Two-particle interferometry (HBT) is the most direct probe of the space–time structure of the hot system formed in high energy heavy ion collisions [8,9], and, along with flow, the past decades have also seen extensive pion HBT measurements map out the space–time geometry and dynamics of heavy ion collisions from Bevalac (∼ 1 A GeV) energies [10], at the AGS (2–11 A GeV) [11–14], and to the highest available energy (∼ 160 A GeV) at the CERN SPS [15,16]. With the exception of some recent preliminary studies [12,14,17], all experimental studies to date have assumed cylindrical symmetry of the source about the beam axis, strictly valid only for \( b = 0 \).

The E895 Collaboration at the AGS used a large acceptance detector to measure charged particles from Au + Au collisions. For every event, the charged particle multiplicity is used to estimate the magnitude of the impact parameter vector \( |b| \), with an uncertainty ∼ 0.5 fm. For all data presented here, the estimated impact parameter range is \( |b| = 4–8 \) fm. The direction of \( b \) is determined from the azimuthally-anisotropic momentum distribution of protons and light nuclear fragments (not pions), with an estimated uncertainty of ∼ 20–35°. Further details are available elsewhere [18,19].

Experimental details, such as correction for momentum resolution and particle identification, of the (azimuth-integrated) HBT analysis have been published [11,12]. Here, we discuss points particular to the azimuthally-sensitive analysis.

As in “standard” HBT analyses, correlation functions are constructed by dividing the two-pion yield as a function of relative momentum \( q = p_1 - p_2 \) by a background generated by mixing pions of the current event with those of previous events [20]. As is well-known, single-particle acceptance/efficiency and phase-space effects cancel out with this “event-mixing” technique. However, there is a special consideration here. In the present analysis, we generate correlation functions with selections on the pair angle with respect to the reaction plane \( \phi = \angle(\mathbf{K}_\perp, \mathbf{b}) \), where \( \mathbf{K}_\perp = (p_1 + p_2)_\perp \) is the total momentum of the pair perpendicular to the beam [21]. Thus, we only mix events which have similar (within 5°) directions of reconstructed \( \mathbf{b} \).

The correlation function was binned according to \( q \)-components in the Bertsch–Pratt (“out-side-long”) decomposition as measured in the Au + Au c.m. frame [22]. In this Letter, we use the subscripts “o, s, l” to stand for “out, side, long”. Here, \( q_l \) is the component parallel to the beam, \( q_o \) is the component parallel to \( \mathbf{K}_\perp \), and \( q_s \) is perpendicular to \( q_l \) and \( q_o \). Particles in a pair were ordered such that \( q_l > 0 \); the signs of the other two components were retained.

Especially in the \( q_o–q_l \) and \( q_s–q_l \) projections, the correlation functions display a distinct “tilt”, which evolves with the emission angle \( \phi \). As discussed below, this tilt arises from fundamental geometry [6], in contrast to a similar tilt observed [16] in some \( q_o–q_l \) projections measured far from midrapidity for azimuthally-integrated HBT, which arises from dynamic (longitudinal flow) effects [23]. Aside from these dynamical effects, there are no two-dimensional tilts in azimuthally-integrated HBT, and only one-dimensional projections are typically shown [12].

Using a maximum-likelihood technique, we fit the correlation functions for each of the \( \phi \) bins with the standard Gaussian parameterization [9]

\[
C(q, \phi) = 1 + \lambda(\phi) \exp\left[ -\sum_{i,j=0,1} q_i q_j R_{ij}^2(\phi) \right].
\] (1)

In contrast to azimuthally-integrated analyses, all six radius parameters are relevant here [5,6,9].

Two-dimensional projections of the fits, weighted according to the mixed-event background, are shown as contours in Fig. 1. Since the relative sign of any component of \( q = p_1 - p_2 \) is arbitrary (while the relative sign between components is not arbitrary), the two-dimensional projections in Fig. 1 have been reflected for clarity. However, the fits were performed...
Fig. 1. Two-dimensional projections in $q_o-q_s$ (left column), $q_o-q_l$ (center column), and $q_s-q_l$ (right column), of the correlation functions measured for midrapidity pions from semi-central Au+Au collisions at 4 A GeV. For each projection, the unplotted component is integrated over ±30 MeV/c. Indicated by the labels on the right, projections are shown for emission angles with respect to the reaction plane $D_{45^{\circ}}$: $5^\circ$ (top row) to $D_{360^{\circ}}$: $5^\circ$ (bottom row). Solid lines show projections of fits with Eq. (1).

in the full three dimensions, with each pion pair being weighted only once.

Although the full structure of the correlation function is only visible in multi-dimensional projections such as those in Fig. 1, one-dimensional projections are commonly shown for azimuthally-integrated HBT analyses, to provide a feeling for the quality of the data and fit. For reference, one-dimensional projections of the correlation function and fits, for one $\phi$ slice, are shown in Fig. 2.

The seven fit parameters $R_{ij}$ and $\lambda$ are plotted as a function of $\phi$ in Fig. 3. As is well-known [8,9], the HBT “radii” parameters are in general not strictly the radii of the emitting source, but have a geometrical interpretation as lengths of homogeneity of the emitting region. For realistic sources, various interpretations of $\lambda$ have been proposed (see, e.g., [24,25]). At AGS energies, the observed strong energy dependence of $\lambda$ was well reproduced [11] by detailed simulations using the RQMD (v2.3) transport model [26]; there, the decrease of $\lambda$ with energy was attributed to increased production of long-lived $\pi^-$-emitting particles at higher energy (see also [27]). In this scenario then, the HBT radii characterize a Gaussian “core” of directly-produced pions, with $\langle 1$ resulting from a well-separated “halo” of pions emitted from long-lived particles [28]. In this framework, we now discuss the $\phi$-dependence of the HBT parameters, and how to extract the underlying geometry of the “core” of the source.

The $\phi$-independence of $\lambda$ indicates that the fraction of $\pi^-$ from long-lived resonances is independent of emission angle with respect to the reaction plane. This seems reasonable, given the relatively weak anisotropic flow at these energies [18], as well as the
Fig. 1. Correlation function projections of each quantify the tilt observed in the two-dimensional equations \[5,6\] the reaction plane, and away from the beam axis \[6\]. ellipsoidal source which is tilted in coordinate space in spheres \[3,12\].

randomizing effect on the pion from decay of the parent.

\[R_o\] and \(R_s\) show significant equal and opposite second-order oscillations in \(\phi\), consistent with the simple picture of a transverse profile of the pion-emitting region that reflects the “almond-shaped” overlap region (with a larger spatial extent perpendicular to \(b\) than parallel to \(b\)) between the target and projectile spheres \[3,12\].

Note that the “cross-term” radii \(R_{o1}^2, R_{o3}^2, R_{s1}^2\) quantify the tilt observed in the two-dimensional correlation function projections of each \(\phi\)-bin of Fig. 1. \(R_{o1}^2\) and \(R_{s1}^2\) display equal-magnitude first-order oscillations, consistent with pion emission from an ellipsoidal source which is tilted in coordinate space in the reaction plane, and away from the beam axis \[6\].

In general, the six HBT radii are related to the spatiotemporal structure of the emitting source via the equations \[5,6\]

\[
\begin{align*}
R_s^2 & = S_{11} \sin^2 \phi + S_{22} \cos^2 \phi - S_{12} \sin 2\phi, \\
R_o^2 & = S_{11} \cos^2 \phi + S_{22} \sin^2 \phi + S_{12} \sin 2\phi
\end{align*}
\]

\[
-2\beta_1 S_{01} \cos \phi - 2\beta_2 S_{02} \sin \phi + \beta_1^2 S_{00},
\]

\[
R_i^2 = S_{33} - 2\beta_i S_{03} + \beta_i^2 S_{00},
\]

\[
R_{\alpha i}^2 = S_{12} \cos 2\phi + \frac{1}{2} (S_{22} - S_{11}) \sin 2\phi + \beta_1 S_{01} \sin \phi - \beta_1 S_{02} \cos \phi,
\]

\[
R_{\alpha i}^2 = (S_{13} - \beta_i S_{01}) \cos \phi - \beta_\perp S_{03} + (S_{23} - \beta_i S_{02}) \sin \phi + \beta_i \beta_\perp S_{00},
\]

\[
R_i^2 = (S_{23} - \beta_i S_{02}) \cos \phi - (S_{13} - \beta_i S_{01}) \sin \phi.
\] (2)

where \(\beta_\perp\) and \(\beta_i\) are the average pair velocities in the transverse and longitudinal direction, and the spatial correlation tensor \(S_{\mu\nu}\) is given by

\[
S_{\mu\nu} = \langle \hat{x}_\mu \hat{x}_\nu \rangle, \quad \hat{x}_\mu = x_\mu - \bar{x}_\mu,
\]

\[
\mu, \nu = 0, 1, 2, 3,
\] (3)

where the brackets \(\langle \rangle\) indicate an average over the emitting source. In Eqs. (2), \(\mu = 3\) refers to the beam direction, and \(\mu = 1\) is parallel to \(b\).

Simultaneously fitting the six \(R_i^2(\phi)\) of Fig. 3 with Eqs. (2) (treating the \(S_{\mu\nu}\) as \(10\) \(\phi\)-independent fit parameters) allows extraction of the full spatial correlation tensor

\[
S = \begin{pmatrix}
13.4 \pm 3.5 & 0.7 \pm 0.9 & 0.2 \pm 1.0 & 0.0 \pm 1.1 \\
0.7 \pm 0.9 & 21.3 \pm 0.9 & -0.1 \pm 0.6 & 4.5 \pm 0.6 \\
0.2 \pm 1.0 & -0.1 \pm 0.6 & 25.5 \pm 1.0 & -0.3 \pm 0.6 \\
0.0 \pm 1.1 & 4.5 \pm 0.6 & -0.3 \pm 0.6 & 22.8 \pm 0.9
\end{pmatrix}
\]

where all units are in fm\(^2\) and \(c = 1\). The fits are shown by solid lines in Fig. 3.

The only significantly non-vanishing tensor elements are the diagonal ones \(S_{00}\) and \(S_{13}\). This fact, along with observations of identical oscillation amplitudes in \(R_o^2\) and \(R_s^2\), and in \(R_{o1}^2\) and \(R_{s1}^2\), support the assumption of \(\phi\)-independence of \(S_{\mu\nu}\) in the fit \[5,6\].

\(S_{13}\) and \((S_{22} - S_{11})\) quantify, respectively, first and second harmonic oscillations of the HBT radii with respect to the measured reaction plane. However, due to finite particle multiplicity, the measured reaction plane differs statistically from the true one by some angle \(\Delta\phi\) \[18,29\]. This reaction plane dispersion results in an apparent reduction in both \(S_{13}\) and \((S_{22} - S_{11})\) (but does not affect quantities \(S_{00}\), \(S_{33}\), and \((S_{11} + S_{22})\)). According to \[5,29,30\]

\[
S_{13,m} = S_{13} \cdot \langle \cos(\Delta\phi) \rangle
\] (4)

\[
(S_{22} - S_{11})_m = (S_{22} - S_{11}) \cdot \langle \cos(2\Delta\phi) \rangle.
\] (5)

\[b = 4-8\ \text{fm}, \quad -0.6 < \gamma_{em} < 0.6, \quad p_T = 0-0.4\ \text{GeV/c}\]
Table 1
First and second harmonic reaction plane dispersion correction factors for the three datasets discussed

<table>
<thead>
<tr>
<th>$E$ (A GeV)</th>
<th>$\cos(\Delta \phi)$</th>
<th>$\cos(2\Delta \phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.940</td>
<td>0.787</td>
</tr>
<tr>
<td>4</td>
<td>0.853</td>
<td>0.584</td>
</tr>
<tr>
<td>6</td>
<td>0.724</td>
<td>0.384</td>
</tr>
</tbody>
</table>

where $S_{13,m}$ is the observed value, and $S_{13}$ is the “true” value. The correction factors obtained by following the procedure outlined in Refs. [5,18,19,29] are listed in Table 1. Hence, for the 4 A GeV data, the reaction plane-dispersion corrected values are $S_{11} = 19.8 \pm 1.2$ fm$^2$, $S_{22} = 27.0 \pm 1.4$ fm$^2$, and $S_{13} = 5.2 \pm 0.7$ fm$^2$.

The spatial tilt of the emission ellipsoid in coordinate space is given by

$$\theta_s = \frac{1}{2} \tan^{-1} \left( \frac{2S_{13}}{S_{33} - S_{11}} \right) = 37^\circ \pm 4^\circ. \quad (6)$$

Rotating the spatial correlation tensor (corrected for reaction-plane dispersions) by this angle returns the diagonal tensor

$$R^T_3(\theta_s) \cdot S \cdot R_3(\theta_s)$$

whose diagonal elements are the squared lengths of homogeneity ($S_{\mu \nu} = \sigma^2_{\mu}$) in the $t$, $x$, $y$, $z$ directions.

Similar results are obtained at other energies. Figs. 4 and 5 show the HBT parameters obtained at 2 and 6 A GeV [31,32]. Especially at the highest energy, degraded reaction-plane resolution significantly reduces the second-order oscillations in the transverse radii. Considered in isolation, $R^2_3$ in Fig. 5 may be fit by a constant ($\phi$-independent) value as well as anything else. However, the simultaneous fit of all $R^2_3(\phi)$ allows relatively clean extraction of the source shape parameters. Reaction plane dispersion corrections are applied to all data, but the effects are small. For the worst case (6 A GeV dataset), the corrections result in a decrease (increase) in $\sigma_x$ ($\sigma_y$) of 0.3 fm, and $5^\circ$ reduction in $\theta_s$; $\sigma_c$ and $\sigma_r$ are unchanged.
Table 2

Lengths of homogeneity $\sigma_{x}$ and tilt angle at each collision energy. In addition to experimental results, predictions of the RQMD model with meanfield off (cs) and on (mf) are given.

<table>
<thead>
<tr>
<th>$E$ (A GeV)</th>
<th>$\sigma_{t}$ (fm/c)</th>
<th>$\sigma_{x}$ (fm)</th>
<th>$\sigma_{y}$ (fm)</th>
<th>$\sigma_{z}$ (fm)</th>
<th>$\theta_{z}$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Data</td>
<td>3.5 ± 1.0</td>
<td>4.2 ± 0.2</td>
<td>5.8 ± 0.2</td>
<td>5.4 ± 0.1</td>
<td>47 ± 5</td>
</tr>
<tr>
<td>RQMD cs</td>
<td>6.6 ± 0.2</td>
<td>2.9 ± 0.1</td>
<td>4.4 ± 0.1</td>
<td>4.4 ± 0.1</td>
<td>49 ± 2</td>
</tr>
<tr>
<td>RQMD mf</td>
<td>5.4 ± 0.2</td>
<td>3.1 ± 0.1</td>
<td>5.0 ± 0.1</td>
<td>4.6 ± 0.1</td>
<td>64 ± 2</td>
</tr>
<tr>
<td>4 Data</td>
<td>3.7 ± 0.5</td>
<td>4.0 ± 0.1</td>
<td>5.2 ± 0.1</td>
<td>5.2 ± 0.1</td>
<td>37 ± 4</td>
</tr>
<tr>
<td>RQMD cs</td>
<td>6.4 ± 0.2</td>
<td>3.3 ± 0.1</td>
<td>4.2 ± 0.1</td>
<td>4.7 ± 0.1</td>
<td>33 ± 3</td>
</tr>
<tr>
<td>RQMD mf</td>
<td>4.6 ± 0.3</td>
<td>3.5 ± 0.1</td>
<td>4.7 ± 0.1</td>
<td>4.7 ± 0.1</td>
<td>45 ± 3</td>
</tr>
<tr>
<td>6 Data</td>
<td>3.8 ± 0.5</td>
<td>3.5 ± 0.2</td>
<td>4.8 ± 0.2</td>
<td>4.7 ± 0.1</td>
<td>33 ± 6</td>
</tr>
<tr>
<td>RQMD cs</td>
<td>5.9 ± 0.2</td>
<td>3.5 ± 0.1</td>
<td>4.3 ± 0.1</td>
<td>4.9 ± 0.1</td>
<td>28 ± 3</td>
</tr>
<tr>
<td>RQMD mf</td>
<td>5.0 ± 0.3</td>
<td>3.6 ± 0.1</td>
<td>4.7 ± 0.1</td>
<td>4.5 ± 0.1</td>
<td>48 ± 5</td>
</tr>
</tbody>
</table>

The reaction plane is longer than the axis in the reaction plane (the “almond” shape referred to above). The extension in the temporal direction is consistent with observations at high energy [11,13,15,16].

The observed transverse “almond” shape is reminiscent of the entrance channel geometry. In the simplest picture, pions are emitted from the overlap region of the two spherical Au nuclei. For impact parameter $b = 4$ fm (8 fm), the spatial RMS of the overlap region is 2.2 fm (1.3 fm) parallel to $b$, and 2.9 fm (2.4 fm) perpendicular [33]. While the linear scale of the freezeout distribution is roughly twice these estimates (see Table 2), indicating significant expansion, the aspect ratio $\sigma_{y}/\sigma_{x} \approx 1.3$–1.4, is in the range (1.3–1.9) expected from this naive picture.

On the other hand, the large positive tilt is a geometric feature of the collision dynamics [6]. Studies with the RQMD model indicate that most of the low-$p_{T}$ pions arise from $\Delta$ decay, so it is not surprising that the tilted freeze-out distribution resembles the baryonic distribution in coordinate space calculated in transport codes [4].

Experimental access to this level of geometric detail on the freezeout distribution is unprecedented, and represents an exciting new opportunity to study the dynamical response of hot nuclear matter to compression. A detailed theoretical discussion is beyond the scope of this Letter, but we note that an identical analysis on pions generated by the RQMD model displays considerable sensitivity on the dynamical effect of the nuclear meanfield; RQMD values are listed in Table 2. Although the spatial scale is underpredicted at the lower energies and the temporal scale overpredicted (noted already for central collisions [11]), qualitatively, the model reproduces the “almond” shape and large positive tilt angles remarkably well. Since the model better describes proton directed flow when the mean field is included in the calculation [19], it is interesting to note that the tilt angles (the spatial counterpart of proton directed flow [6]) reproduce observation better when the mean field is off.

We stress that these coordinate-space anisotropies represent new information, independent of momentum-space anisotropies (directed and elliptical flow). Momentum-space tilt angles (flow angles [34]) at these energies are only a few degrees [19], and, indeed, the directed flow (momentum-space tilt) of pions at these energies is in the negative direction [6,35,36], opposite the coordinate-space tilt $\theta_{z}$. Experimental information on the interplay between coordinate and momentum space anisotropies should help resolve theoretical issues, such as the coexistence of flow and antiflow components at these energies [4].
Further studies of the elliptical transverse shape of the source at higher energy should prove quite interesting. It is known, for example, that the elliptic flow of nuclear matter (protons and pions) in momentum space changes sign at AGS energies \[18,36–38\], from negative (more momentum out-of-plane) at low collision energy to positive. The coordinate-space analogue of negative elliptic flow is the “almond” shape we observe. The degree to which this shape follows the momentum space and evolves to an in-plane shape at high energies, may provide valuable information on the detailed nature and cause of elliptic flow \[39\]; RQMD predicts \[40\] that the “almond” shape is retained even at RHIC energies.

In conclusion, we have presented the first full measurement of the azimuthal dependence of pion interferometry. For semi-peripheral \(\text{Au}+\text{Au}\) collisions at 2–6 \(\text{A GeV}\), the spatial correlation tensor, \(S_{\mu\nu}\), extracted from the \(\phi\)-dependences of the HBT radius parameters, reveals an ellipsoidal pion emission region with an “almond” transverse profile and which is strongly tilted in the reaction plane away from the beam axis. Consistency relations indicate that the extracted geometry is negligibly affected by possible transverse space-momentum correlations. The RQMD transport model reproduces the qualitative features of the data quite well, and reveals a dependence of the coordinate-space anisotropies on the action of the nuclear meanfield. Through application of this new type of analysis at other energies and careful theoretical comparisons, it is hoped that fresh insight on the dynamics of non-central heavy ion collisions may be gained.

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References

J. Brachmann et al., nucl-th/9912014.
[21] Selecting instead on $p_{T} > 1$, yields results similar to those we discuss. However, one loses pair statistics from pairs which straddle cut borders in $b$.
[31] For the 2 AGeV analysis, our rapidity cut is centered somewhat forward of midrapidity, for statistics and acceptance reasons; hence, $S_{03}$ does not vanish [23], resulting in a net offset in $R_{2}^{ol.}(b)$.
[32] In the 6 AGeV case, the fit produces an additional non-zero component: $S_{02} = 1.3 \pm 1.0$. This results in the slight asymmetry that is clear in the fit lines to $R_{2}^{ol.}$ and $R_{2}^{os}$. We do not consider significant this barely-nonvanishing component.
[33] Recall that the RMS values are significantly less than the hard-sphere radius of 7 fm for a uniformly-filled hard sphere of radius $R$, the RMS in any one direction is $R/\sqrt{3}$.
[40] J. Yang et al., submitted.