Problem Set 5  
Due May 23, 2006


3) Follow the example of lecture 6, page 6 where we found the LSF for the slope of the line (b), \(y=1+bx\), and show (using the data in the table on page 6) that \(b=1.05\).

4) A set of \(n\) data points \((x_i, y_i \pm \sigma_i)\) are related by: \(y=A+5x\).
   a) Use the method of Least Squares to show that the best estimate of the intercept, \(A\), is given by:
      \[
      A = \frac{\sum_{i=1}^{n} y_i / \sigma_i^2 - 5 \sum_{i=1}^{n} x_i / \sigma_i^2}{\sum_{i=1}^{n} 1 / \sigma_i^2}
      \]
   b) Use propagation of errors to show that the variance of \(A\) is given by:
      \[
      \sigma_A^2 = \frac{1}{\sum_{i=1}^{n} 1 / \sigma_i^2}
      \]

5) A certain theory states that the angular distribution of the decay of an unstable particle should have a probability distribution function of the form:
   \[
   p(\cos \theta) = N(1 + \beta \cos^2 \theta)
   \]
   Here both \(N\) and \(\beta\) are constants. An experiment measures ten examples of the decay of this unstable particle and finds the following values of \(\cos \theta\): \((-0.15, -0.25, -0.35, -0.5, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95)\). We wish to determine the value of \(\beta\) using the Maximum Likelihood Method.
   a) Use the normalization condition for a probability distribution function to show that:
      \[
      N = \frac{1}{2(1 + \beta / 3)}
      \]
      For this problem the limits on \(\cos \theta\) are \([-1, 1]\).
   b) Write down the Likelihood Function for this problem.
   c) Make a plot of the Likelihood Function vs. \(\beta\) for \(-1.2 < \beta < 1.2\). Use this plot to find the value of \(\beta\) that maximizes the Likelihood Function. You may want to write a program to do this part!