Chapter 31: Induction and Inductance

In Ch 30 we learned the following about magnetic fields:

a) A magnetic field can exert a force on a current carrying wire
b) If the wire is a closed loop then the magnetic field can exert a torque on the loop.
   As a result of the torque the loop will rotate. This is the basis of the electric motor.
c) A wire carrying a current can generate a magnetic field.

We also learned that an electric field can cause a current to flow. It is reasonable to ask:
Can a magnetic field cause a current to flow?

The answer is YES!

And Faraday’s Law of Induction describes the conditions necessary for a magnetic field to induce a EMF.

There are three classic demonstrations of a magnetic field causing a current to flow:

1) Take a conducting loop of wire that initially has zero current flowing and turn it by hand in the presence of a B-field. A current will be generated in the loop!
2) Take a conducting loop of wire that initially has zero current flowing and take bar magnet and move it through the loop. A current will be generated in the loop!
3) Take a conducting loop of wire that initially has zero current flowing and put it next to another loop of wire. We then flow current (or change the amount of current flowing) in the 2nd loop. We find that while the current is changing in the 2nd loop current will flow in the 1st loop.

What is common in these 3 experiments is that a changing magnetic field induces a current.

Faraday’s Law of Induction: When the number of magnetic field lines is changing through a conducting loop a current is induced in the loop.
Faraday’s Law of Induction

In both situations a current will be induced in a loop due to a changing magnetic field. While Faraday’s Law connects an induced current (or EMF) to a changing magnetic field it doesn’t tell us how much EMF to expect. We can calculate the expected EMF using a quantity called MAGNETIC FLUX, $\phi_B$. The magnetic flux is defined by:

$$\phi_B = \int \overline{B} \cdot d\overline{A}$$

The unit of magnetic flux is: Tesla-m²=weber

The magnitude of the induced EMF in a conducting loop depends on how fast the B-flux changes:

$$E = -\frac{d\phi_B}{dt} = -\frac{d}{dt} \int \overline{B} \cdot d\overline{A}$$

The minus sign is necessary because the induced emf (and current) tends to oppose the change in the magnetic flux.

If the current consists of N loops then E is:

$$E = -\frac{N d\phi_B}{dt}$$
Magnetic Flux

The key to Faraday’s Law of Induction is that the magnetic flux is changing over time. Magnetic Flux is basically the product of the magnetic field and an area.

A change in the magnetic flux can be caused by:

1) A magnetic field that is changing over time.
   We can do this by varying the current in a wire as a function of time or moving a bar magnet near a wire loop.

2) An area with is changing over time.
   We can do this by rotating a wire loop in a magnetic field (HRW 31-10) or moving a loop in/out of a magnetic field (HRW 31-41).

3) A magnetic field AND an area changing with time.

\[ \phi_B = \int \overline{B} \cdot d\overline{A} \]

Note: It is possible to have a magnetic field that is changing with time and an area that is changing with time BUT the product of the two is constant with time!
Faraday’s Law of Induction continued

Example: A UHF television antenna in the shape of a loop has a diameter of 11 cm. If the magnetic field of a TV signal is perpendicular (normal) to the plane of the loop and at an instant in time is changing at the rate of 0.16T/s. What is the EMF in the antenna?

Since B is perpendicular to the loop the magnetic flux is:

\[ \phi_B = BA \]

with A=area of the loop (A=\(\pi(5.5 \times 10^{-2}m)^2\))

The magnetic flux is changing as a function of time because the B-field is changing:

\[ \frac{d\phi_B}{dt} = A \frac{dB}{dt} = (\pi(5.5 \times 10^{-2}))^2 (0.16 T/s) = 1.5 \times 10^{-3} V \]

Therefore the induced EMF is:

\[ E = -\frac{d\phi_B}{dt} = -1.5 \times 10^{-3} V \]

The minus sign has to do with the direction of E

Example: A small loop of area A is inside of and has its axis in the same direction as a long solenoid of n turns per unit length and current I. If I=I_0\sin(\omega t), find the emf in the loop.

The magnetic field in the solenoid is: \(B=\mu_0 n I=\mu_0 n I_0 \sin(\omega t)\)

The magnetic flux is:

\[ \phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos \theta dA = \int B(1)dA = BA = (\mu_0 n I \sin \omega t)A \]

The induced EMF is:

\[ E = -\frac{d\phi_B}{dt} = -\frac{d}{dt} (\mu_0 n I \sin \omega t)A = -\mu_0 n I \omega A \cos \omega t \]

In this example the EMF is varying as a function of time.
Lenz’s Law

Lenz’s Law allows us to figure out the direction of the induced current in a loop.

“An induced current has a direction such that the B-field due to the induced current opposes the change in the magnetic flux that induces the current.”

Example: A loop of wire enters a region of uniform magnetic field that points out of the page.

What is the direction of the induced current in the loop?

As the loop enters the region with B-field its magnetic flux will be increasing. Lenz’s Law that the loop wants to set up a B-field that opposes this increase. Thus the induced B-field must be into the page. If the B-field is into the page then the current in the loop must be going CLOCKWISE.
Force and Power

In the figure on the right Mr. Hand is trying to pull a loop of wire out of the region with magnetic field. If the loop is pulled at constant speed how much force does Mr. Hand have to exert? To calculate the force exerted on the loop due to the induced current we need to know the direction of this current and its magnitude. Lenz’s law gives the direction:

As we pull the loop the amount of flux it encloses is decreasing. Lenz’s law says that induced current should set up a field to oppose this decrease. Therefore the induced current wants to increase the flux by increasing the B-field. This induced field must be in the same direction as the original B-field, into the page. Using the 2nd RHR we see that the current must flow in the clockwise direction.

The force on the wire can be calculated from: $\vec{F} = I\vec{L} \times \vec{B}$

The length vector (L) is in the direction of the current. There are 3 wire segments in the B-field. The forces on the top and bottom wire pieces (2, 3) are in opposite directions so they cancel. The force on piece 1 is: $|\vec{F}_1| = ILB \sin \theta = ILB \sin(90) = ILB$

If we assume that the wire has resistance R we can calculate the induced current I using:

$I = \frac{E}{R}$

$E = \frac{d\phi}{dt} = \frac{d}{dt} \int B \cdot dA = \frac{d}{dt} \int B dA \cos \theta = \frac{d}{dt} \int B dA \cos 0 = \frac{d}{dt} \int B dA = \frac{d}{dt} B \int dA = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv$

$F_1 = \frac{B^2L^2v}{R}$
Force and Power continued

This force can be quite large!

Example: We have a loop made out of thick copper wire. The loop is a square 1m per side and the wire has a diameter of 10 cm. Suppose this loop is in a 1T B-field and we move it at \(v=1\) m/s. How much force do we have to exert?

We need to know the resistance of the wire. Since we know it is made of copper and we know its dimensions we can calculate its resistance using:

\[
R = \frac{\rho_{copper} L}{A} = \frac{(1.7 \times 10^{-8} \Omega \text{m})(4m)}{\pi(0.05m)^2} = 8.6 \times 10^{-6} \Omega
\]

The force is:

\[
F = \frac{B^2 L^2 v}{R} = \frac{(1T)^2 (1m)^2 (1m/s)}{8.6 \times 10^{-6} \Omega} = 1.2 \times 10^5 N \approx 12 \text{ tons}
\]

We can calculate the power dissipated in the wire using:

\[
P = I^2 R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2 L^2 v^2}{R}
\]

For our problem the power dissipated is:

\[
P = \frac{B^2 L^2 v^2}{R} = \frac{(1T)^2 (1m)^2 (1m/s)^2}{8.6 \times 10^{-6} \Omega} = 1.2 \times 10^5 W
\]

Most likely, the wire would melt!
A last example

Two straight conducting rails form a right triangle. A conducting bar in contact with the rails starts at the vertex at \( t=0 \). The bar moves with \( v=5.2 \text{ m/s} \). A magnetic field, \( B=0.35 \text{T} \) is pointing out of the page.

1) What is magnetic flux as a function of time?

\[
\phi = \int \overrightarrow{B} \cdot d\overrightarrow{A} = \int B dA \cos \vartheta = \int B dA \cos 0 = B \int dA = BA
\]

We need to find the area of the triangle \( (A) \) vs time. Since the rails form a right triangle the height of the triangle = \( \frac{1}{2} \) the base.

The height of the triangle, \( h \), is: \( h=vt \).

The area of the triangle is: \( \frac{1}{2}(\text{base})(\text{height})=\frac{1}{2}(2h)(h)=h^2=(vt)^2 \)

The magnetic flux is: \( BA=Bv^2t^2=(0.35\text{T})(5.2\text{m/s})^2t^2=9.5t^2 \)

2) What is the induced EMF vs time?

\[
E = \frac{d}{dt} \phi = \frac{d}{dt} (9.5t^2) = 19t \ (\text{Volts})
\]

At \( t=1 \text{ sec} \) \( E=19\text{V} \), at \( t=5 \text{ sec} \) \( E=90\text{V} \).

3) What is the direction of the induced current?

Lenz’s law says we want to oppose the change in flux. Since the flux is increasing our induced current will set up a \( B \)-field to oppose the increase. Therefore the induced magnetic field will be INTO the page. Thus the induced current will be CLOCKWISE.