(4.1) Griffiths 5.1. Consider the center of mass and relative coordinates (i.e., \( \vec{r} = \vec{r}_1 - \vec{r}_2 \)). Find expressions for the center of mass momentum \( \vec{P} \) and the momentum corresponding to the relative coordinates (denote it by \( \vec{p} \)), in terms of \( \vec{p}_1 \) and \( \vec{p}_2 \). Show that \( \vec{P} \) commutes with \( \vec{r} \). Write the total angular momentum \( \vec{L}_{tot} \) in terms of the angular momenta of the two particles. Decompose \( \vec{L}_{tot} \) into the angular momentum of the center of mass \( \vec{L}_{cm} \) and the angular momentum of the relative coordinates \( \vec{L} \). The total Hamiltonian is

\[
H_{tot} = \frac{\vec{p}_1 \cdot \vec{p}_1}{2m_1} + \frac{\vec{p}_2 \cdot \vec{p}_2}{2m_2} + V(\vec{r}_1 - \vec{r}_2).
\]

Write \( H_{tot} \) in terms of the center of mass and relative variables. You may (if you wish) use this form to write down the time-independent Schrödinger equation and thus do part (b). Find as many observables as you can which commute with the total Hamiltonian if \( V \) depends only on \( |\vec{r}| \). (6 points)

(4.2) Griffiths 6.14 (4 points)

(4.3) Griffiths 6.33 (3 points)

(4.4) Griffiths 6.34. You may need to remind yourself of elementary electrostatics from Halliday and Resnick. (7 points).

(4.5) Consider a spin \( S = 1 \) particle which is in an orbital angular momentum state with \( L = 2 \). What are the allowed values of \( J \)? Count the number of states in the \( J, J_z \) basis and the \( L_z, S_z \) basis and verify that they are equal. The Hamiltonian of the particle is given by

\[
H = -\frac{\lambda}{\hbar^2} \vec{L} \cdot \vec{S}
\]

where \( \lambda > 0 \). Find the energy eigenvalues (we ignore the kinetic energy and other interactions for simplicity) and the degeneracy of each level. (5 points)

(4.6) Consider a symmetric top, i.e., a rigid body with principal moments of inertia \( I_x = I_y \neq I_z \). Recall that the rotational kinetic energy of a free rigid body in classical
physics is given in general by $(I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2)/2$. Write the Hamiltonian (you may ignore translational motion) in terms of angular momentum operators; we did this last quarter. Find the eigenvalues of the Hamiltonian for general (integral) values of the angular momentum. So far this is a review of angular momentum algebra from last quarter. Now consider a slightly asymmetric top with $I_x = I + (\Delta/2)$ and $I_y = I - (\Delta/2)$. To first order in $\Delta$ calculate the corrections to the energies for the case of angular momentum 1, i.e., $L = 1$. This problem appeared in a first year graduate quantum mechanics examination in a reputable university. (6 points)