Chapter 3

Kinematics in Two Dimensions:

Projectile motion
Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at 9.80 m/s$^2$.

\[ a_y = -9.80 \text{ m/s}^2 \quad a_x = 0 \]

\[ v_x = v_{ox} = \text{constant} \]
**Projectile Motion**

**Example:** The Height of a Kickoff

A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.
Projectile Motion

\[ v_{oy} = v_o \sin \theta = (22 \text{ m/s}) \sin 40^\circ = 14 \text{ m/s} \]

\[ v_{ox} = v_o \cos \theta = (22 \text{ m/s}) \cos 40^\circ = 17 \text{ m/s} \]
Projectile Motion

\[ y \quad a_y \quad v_y \quad v_{oy} \quad t \]

<table>
<thead>
<tr>
<th>y</th>
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<th>( v_y )</th>
<th>( v_{oy} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
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<td>0</td>
<td>14 m/s</td>
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### Projectile Motion

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\[ v_y^2 = v_{oy}^2 - 2gy \quad \Rightarrow \quad y = \frac{v_y^2 - v_{oy}^2}{-2g} \]

\[ y = \frac{0 - (14 \text{ m/s})^2}{-2(9.8 \text{ m/s}^2)} = +10 \text{ m} \]
**Projectile Motion**

**Example:** The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?
Projectile Motion

\[
\begin{align*}
\text{\(v_{0y}\)} & \quad \theta \\
\text{\(v_0\)} & \\
\text{\(+y\)} & \\
\text{\(+x\)} & \\
\text{\(H = \text{Maximum height}\)} & \\
\text{\(R = \text{Range}\)} & \\
\text{\(a_y\)} & \text{-9.80 m/s}^2 \\
\text{\(v_y\)} & \text{14 m/s} \\
\text{\(v_{0y}\)} & \\
\text{\(t\)} & \\
\end{align*}
\]
### Projectile Motion

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\[
y = v_{oy}t - \frac{1}{2}gt^2
\]

\[
0 = (14 \text{ m/s})t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2
\]

\[
0 = 2(14 \text{ m/s}) - (9.80 \text{ m/s}^2)t
\]

\[
t = 2.9 \text{ s}
\]
Example: The Range of a Kickoff

Calculate the range $R$ of the projectile.

\[ x = v_{ox} t \]

\[ = (17 \text{ m/s})(2.9 \text{ s}) = +49 \text{ m} \]
**Projectile Motion**

**Example: Shoot the falling object.**
A metal can is dropped from a platform at the same time a gun located at some distance from the can and at a lower height tries to shoot it as it falls. If the can is initially a height $h$ above the height of the gun, the gun is a horizontal distance $d$ away from the can, and the muzzle velocity of the bullet is $v_0$, at what angle $\theta$ should the gun be aimed to hit the falling can?
**Projectile Motion**

If $t$ is the time it takes for the bullet to reach the can $\Rightarrow x_{Bullet} = d = v_{0x}t$

$\therefore t = \frac{d}{v_{0x}} = \frac{d}{v_0 \cos \theta}$

y positions at time $t$: $y_{Bullet} = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin \theta)t - \frac{1}{2}gt^2$ and $y_{Can} = h - \frac{1}{2}gt^2$

Condition for the bullet to hit the can at time $t \Rightarrow y_{Bullet} = y_{Can}$

$(v_0 \sin \theta)t - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2 \Rightarrow (v_0 \sin \theta)t = h$

$\therefore (v_0 \sin \theta)\left(\frac{d}{v_0 \cos \theta}\right) = h \Rightarrow \tan \theta = \frac{h}{d} \Rightarrow \theta = \tan^{-1}\left(\frac{h}{d}\right)$

$\therefore$ aim the gun at the initial position of the can to hit it while it is falling
At serve, a tennis player aims to hit the ball horizontally.

a) What minimum speed is required for the ball to clear the \( h = 0.90 \text{ m} \) high net \( q = 15.0 \text{ m} \) from the server if the ball is "launched" from a height of \( w = 2.00 \text{ m} \)?

b) Where will the ball land relative to the player if it just clears the net?

c) How long will the ball be in the air?
**Projectile Motion**

\[ y = v_{oy}t - \frac{1}{2}gt^2 \implies t = \sqrt{\frac{-2(w-h)}{-g}} = \sqrt{\frac{2(1.10)}{9.80}} = 0.474 \text{ s} \]

\[ x = q = v_{\text{min}}t \implies v_{\text{min}} = \frac{q}{t} = \frac{15.0}{0.474} = 31.6 \text{ m/s} \]
**Projectile Motion**

\[ y = v_{oy} t - \frac{1}{2} gt^2 \implies t_{min} = \sqrt{\frac{-2w}{-g}} = \sqrt{\frac{2(2)}{9.80}} = 0.639 \text{ s} \]

\[ x = d_{min} = v_{min} t_{min} = (31.6)(0.639) = 20.2 \text{ m} \]
Projectile Motion

Conceptual Example: Two Ways to Throw a Stone

From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle below the horizontal and stone 2 is thrown at the same angle above the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?
**Projectile Motion**

Initial velocity of stone 1: \( v_0 \) at angle \(-\theta\) from horizontal

\[
\begin{align*}
v_{0x1} &= v_0 \cos(-\theta) = v_0 \cos \theta \\
v_{0y1} &= v_0 \sin(-\theta) = -v_0 \sin \theta
\end{align*}
\]

Initial velocity of stone 2: \( v_0 \) at angle \(+\theta\) from horizontal

\[
\begin{align*}
v_{0x2} &= v_0 \cos \theta \\
v_{0y2} &= v_0 \sin \theta
\end{align*}
\]

Velocity of stone 2 at point \( P \): \( y_2 = 0 \)

\[
v_{x2} = v_{0x2} = v_0 \cos \theta
\]

\[
\begin{align*}
v_{y2} &= v_{0y2} - 2gy_2 = v_{0y2} - 2g \cdot 0 = v_{0y2} \\
v_{y2} &= \pm v_{0y2} = -v_{0y2} = -v_0 \sin \theta
\end{align*}
\]

\[
\therefore v_{0x1} = v_{x2} , \quad v_{0y1} = v_{y2}
\]

→ Both stones strike the water with the same velocity
stone 2 has the same velocity when it is at $P$ as stone 1 has initially, so

$\rightarrow$ both will hit the water with the same velocity (but stone 2 will hit the water later and be horizontally displaced from stone 1)