The goal of this lab is to measure the lifetime ($\tau$) of a radioactive isotope. The lifetime of any radioactive substance is governed by an exponential decay distribution (or law):

$$N(t) = N(0)e^{-t/\tau}$$

In this equation $N(t)$ is the amount of the substance left at time $t$ and $N(0)$ is the amount of the substance at the start of the experiment. The lifetime ($\tau$) governs how fast the substance "disappears". In this experiment we will actually measure the rate of decay $R(t)(=dN/dt)$ of the isotope, i.e. the number of decays per minute as a function of time.

$$R(t) = \frac{dN(t)}{dt} = \frac{N(0)}{\tau}e^{-t/\tau}$$

This equation takes on a simpler form if we take the natural log of both sides:

$$\ln R(t) = \ln\left[\frac{N(0)}{\tau}\right] - \frac{t}{\tau}$$

We now have a linear relationship to work with where the lifetime ($\tau$) is related to the slope of the line.

What's really great about this relationship is that we can determine the lifetime $\tau$ without knowing anything about how much of the substance we actually started out with ($N(0)$)!

In this lab we will measure the lifetime of a meta-stable state of Barium 137 (Ba137). Ba137 is produced in the decay of Cs137 according to the decay chain listed below. Here Cesium undergoes a beta decay (neutron $\rightarrow$ proton + electron + anti-electron neutrino) to an unstable state of Barium. The unstable state of Barium decays via emission of a gamma ray of energy 662 KeV (= 0.662 MeV) with a lifetime of 3.75 minutes.

1) Electronics Preparation:
Since we want to measure the number of decays/minute in this experiment we need to adjust the NaI Spectroscopy Amplifier such that it only counts the unstable Ba137 decays. Since the unstable state of Barium decays via emission of a 662 KeV gamma ray we need to adjust the system so it triggers the computer whenever a gamma ray of this energy range is detected by the NaI crystal. This can be accomplished using the “UL” (Upper Level) and “LL” (Lower Level) controls on the Spectroscopy Amplifier. Input pulses from the NaI detector that are above the voltage level set by the UL control do not send a trigger signal (via the TRG OUT connector) to the computer. Likewise, input signals that are below the level set by the LL control do not trigger the computer. Thus using these controls we can set an energy window that is centered around 662 KeV.

Using the program “MCA with Timer” and a Cs137 source adjust the UL and LL controls such that the program only counts gamma rays that are within $\approx \pm 1\sigma$ of the Cs137 peak. You will have to start and stop the program several times to get the controls set at the proper levels.

2) Sample Preparation:
One must take special care in this experiment because we are dealing with a radioactive liquid. It is important to wear gloves, eye protection, and lab coat during the initial phase of this experiment. Label a test tube with your name and collect ~10 droplets of Ba137. Put a cap on the tube.

3) Data Taking:
Use the program “MCA with Timer” to take the data for the lifetime determination. Before collecting the data, change the numbers under “Total Time of Counting (min)” to 12 and “Duration of Count Time (sec)” to 60. This instructs the program to record the number of counts in 60-second intervals for duration of 12 minutes. Click the “➔” button at the top of the window to start the counting. After 12 minutes, the program asks you for a file name to store the count information. If necessary, you can stop the counting by clicking on the icon that looks like a stop sign.

4) Data Analysis:
a) Use Kaleidagraph to read in the data file and make a semi-log plot of your data. Here the y axis is natural log of the number of counts/minute. For the time values on the x-axis take the midpoint of the time interval (e.g. 0.5 min, 1.5 min etc). Use Kaleidagraph to fit your plot to determine the slope and its error and then calculate the lifetime of Ba137m and its error.

b) Perform a least squares fit to your data to determine the slope and therefore the lifetime of Ba137m. Using the techniques discussed in Lec. 7 to calculate $\alpha_\tau$, the standard deviation of the lifetime. I suggest you write a program for this part. Remember, the function you are fitting to is a straight after you take the natural log of the number of counts/min.

c) Use Kaleidagraph to fit your data to an exponential function of the form:
\[ y(t) = Ae^{-t/B} \]
and determine the lifetime and its error from your fit.

d) The background in your measurements can be taken into account by fitting the uncorrected data to:
\[ y(t) = Ae^{-t/B} + C \]
Use Kaleidagraph to fit this function to your data. Is the $\chi^2$ of the fit reasonable? What is the lifetime and its error? Is your measured lifetime consistent with the known value of $\tau$.

e) Think of a way to measure the background rate. Apply this background correction to your raw data and re-calculate the lifetime using Kaleidagraph as in (c). How does this new value compare with result obtained from the background corrected fitting procedure?

f) Simulation of the lifetime experiment. Here you want to simulate your observed lifetime distribution using the measured $A$, $B$, and $C$ with a program. The procedure is as follow:
   1) generate a random time between 0 and 12 minutes
   2) compute $y(t)$ and the ratio $R = y(t)/y(0)$
   3) generate a random number between 0 and 1
   4) if the random number is less than $R$, accept the event and put it in the appropriate time bin
Repeat the procedure until the accepted number of events is the same as that in the data. Superimpose your simulated data onto the real data and comment on the agreement between the data and simulation.

This method of simulation is often called “acceptance/rejection.” It is a very powerful technique to simulate any probability distribution function but its draw back is that it can be very CPU intensive (i.e. inefficient).