\[
\frac{dI}{dt} = (B(t)^N(t) - \frac{1}{\tau}) I(t)
\]

\[
\frac{1}{N(t)} = \frac{Y(t)}{2\lambda(t)} + \frac{Y(t)}{2\beta(t)} - \frac{Y(t)}{2\beta(t)}
\]

Suppose we use mirror #2 to confine at half.

\[
\frac{d\alpha}{dt} = -\frac{\alpha}{2\lambda(t)}
\]

\[
P_{\text{out}} = \frac{Y(t)}{2\lambda(t)} e^\left(\lambda(t)\sqrt{\tau}\right)
\]

\[
P_{\text{out}} = \left(\frac{Y(t)}{2\lambda(t)}\right) \left[\frac{\lambda(t)Y(t)}{\alpha} - \frac{\tau}{2} \left(\frac{\lambda(t)}{\beta(t)} - 1\right)\right]
\]

\[
= \left(\frac{h_0}{\sigma t}\right) \frac{Y(t)}{2} \sqrt{\frac{\alpha}{\lambda(t)}} \sqrt{\frac{\beta(t)}{\lambda(t)}} \frac{Y(t)}{2\lambda(t)} \\
\text{No.} \quad E_{12} \quad \frac{\alpha}{\lambda(t)} \\
\text{No.} \quad E_{12} \quad \frac{\beta(t)}{\lambda(t)} \text{ No.}
\]

\[
P_{\text{out}} = \frac{\beta(t)}{\lambda(t)} \frac{Y(t)}{2} \left[\frac{\beta(t)}{\lambda(t)} - 1\right]
\]

\[
P_{\text{out}} = \frac{\beta(t)}{\lambda(t)} \frac{I(t)}{2} \left[\frac{\beta(t)}{\lambda(t)} - 1\right]
\]
The "slope efficiency" is defined as
\[ \eta_s = \frac{dP_{out}}{dP} \]

\[ R_p = \frac{P}{A_h \nu} \]

\[ \nu_m = \frac{E_2 - E_0}{h} \]

\[ \eta_s = \frac{Y}{\nu_p} \left( \frac{h \nu_m}{\nu} \right) \frac{A}{A_e} \]

\[ \eta_s = \frac{Y}{\nu_p} \left( \frac{h \nu_m}{\nu} \right) \frac{A}{A_e} \]

\[ \frac{Y}{\nu} \left( \frac{h \nu_m}{\nu} \right) \frac{A}{A_e} \]

\[ \% \text{ of the gain medium filled} \]

\[ \text{Laser quantum efficiency (some of the energy of the excited state is lost: } E_i - E_0) \]

\[ \text{Output coupling efficiency} = \frac{\text{rate of photon output}}{\text{rate of photon lost due to cavity}} \]

Max value = 1.

**3.3.3 CO_2 laser**

Read Example (22)

It puts many things together. It discusses Nd:YAG and YAG laser information that we've assembled about Nd:YAG over the course of the class, including figures and notes that I've enclosed.

In the example: \( N_e = 4 \times 10^{-4} \) \( N_e = 10^{10} \text{ndl concentration} \).
Suppose we only couple out of mirror #2.

$R_2 = 1 \Rightarrow I_{\text{out}} = 0 \Rightarrow Q_{\text{max}}$ is maximum

$R_2 = 0 \Rightarrow I_{\text{out}} = 0$ because $Q = 0$.

$$P_{\text{out}} = A_4 E_2 \frac{V}{2} \left( \frac{P}{P_c} - 1 \right)$$

we expect

$$\frac{dP_{\text{out}}}{dX} = 0 \quad \text{but} \quad P = \frac{V}{P_c} \frac{h V_{\text{ph}}}{T} \frac{A}{T}$$

depends on $Y$ and this on $X$.

... Let $P_{\text{th}} = \text{threshold for } R_2 = 1 \text{ (minimum threshold)}$

$x_n = \frac{P}{P_{\text{th}}}$ so this is a measure of how

far this is from this aim.

$\text{Critically the lower won't lose until}$

$$p_P > P_{\text{th}} = P_{\text{th}} \frac{V_e + \frac{V}{T} + \frac{V}{2}}{V_e + \frac{V}{T} + \frac{V}{2}} \geq 1$$

$$Y = (2Y_0 + Y) \left( \sqrt{x_n^2 - 1} \right) \Rightarrow X = \sqrt{x_n^2 - 1}$$

$$P_{\text{out}} = A_4 E_2 (Y + \frac{Y}{2}) \left( \sqrt{x_n^2 - 1} \right)^2 \text{(7.6.56)}$$
\[ P_{\text{th}} = \frac{1}{2} \left( \psi_{\text{th}} \frac{A}{\nu} \right) Y \]

\[ \psi_{\text{th}} = P_{\text{th}} (Y = 0) \]

\[ P_{\text{th}} = \frac{P_{\text{th}}}{Y} \left( \frac{Y + Y_c}{Y} \right) \]

\[ P_{\text{th}} = \frac{P_{\text{th}}}{Y} \left( \frac{Y}{Y + Y_c} \right) \quad (7.5.1) \]

\[ P_{\text{out}} = A_b \bar{I}_3 \left( \frac{X}{\bar{I}_3} \right) \left( \frac{\frac{Y + Y_c}{Y}}{\frac{Y}{Y + Y_c}} \right) \]

\[ = A_b \bar{I}_3 \left( \frac{Y}{Y + Y_c} \right) \left( s_x \frac{X}{\bar{I}_3 + Y} \right) \left( \frac{Y}{Y + Y_c} \right) \left( \frac{Y}{Y + Y_c} \right) \left( \frac{Y}{Y + Y_c} \right) \]

\[ = \frac{1}{X + Y + Z} = \frac{1}{S + 1} \]

\[ P_{\text{out}} = \left[ A_b \bar{I}_3 \left( \frac{Y}{Y + Y_c} \right) \right] S \left( \frac{X}{S + 1} \right) \quad (7.5.2) \]

\[ \frac{dP_{\text{out}}}{dX_c} = 0 \quad \text{when} \quad \frac{dP_{\text{out}}}{dS} = 0 \]

\[ \frac{dP_{\text{out}}}{dS} = E \left[ \left( \frac{X}{S + 1} \right) + S \left( \frac{-X_c}{(S + 1)^2} \right) \right] \]

\[ X_c (S + 1) - (S + 1)^2 - S X_c = 0 \quad \Rightarrow \quad (S + 1)^2 = X_c \]

\[ P_{\text{out}} = E \left( \sqrt{X_c} - 1 \right) \left( \frac{X_c}{\sqrt{X_c}} - 1 \right) \]

\[ P_{\text{opt}} = \left[ A_b \bar{I}_3 \left( \frac{Y}{Y + Y_c} \right) \right] \left( \sqrt{X_c} - 1 \right) \quad (7.5.6) \]
For $P_n = P_{\text{th}}$ we have:

$$\gamma_{\text{out}} = (2\gamma + \gamma) \left( \sqrt{\frac{\gamma + \frac{\gamma}{\gamma_n}}{\gamma_n}} - 1 \right)$$

$$= 2 \left[ \sqrt{\gamma + \frac{\gamma}{\gamma_n}} - 1 \right]$$

$$\gamma_{\text{out}} = 0$$

$$P_{\text{out}} = \left[ \frac{\gamma + \frac{\gamma}{\gamma_n}}{\gamma_n} - 1 \right]$$

$$P_{\text{out}} = 0$$

The form of $\gamma_{\text{out}}$ and $P_{\text{out}}$ is somewhat deceptive. It's written the way it is for calculational convenience.

But we can learn from it.

Near threshold, $\gamma_{\text{out}}$ and $P_{\text{out}}$ depend sensitively on $\gamma_n$.

But, for far from threshold, things are insensitive.

For far from threshold, things are insensitive.

![Diagram](image)
Close to threshold: \( X \approx 1 \)

\[
X = \frac{P}{P_0} = \frac{P_0}{P_0} \left( 1 + \frac{\frac{x}{2}}{\frac{x}{2}} \right)
\]

\[
\text{let } p = \frac{P}{P_0} (1 + \Delta)
\]

Note: \( S \) must be small for \( X \) to be close to 1.

\[
X = (1 + \Delta)(1 + S) \approx 1 + \Delta + S
\]

At pinch coupling:

\[
Y_{\text{pinch}} = (2 \gamma + \gamma_1) \sqrt{X - 1}
\]

\[
= (2 \gamma + \gamma_1) \left[ \frac{1}{2} (\Delta + S) \right]
\]

\[
S_{\text{opt}} = \frac{1}{2} (\Delta + S_{\text{opt}})
\]

\[
S_{\text{opt}} = \Delta
\]

\[
Y_{\text{opt, pinch}} = (2 \gamma + \gamma_1) \Delta
\]

\[
P_{\text{opt}} = \left[ \frac{1}{2} (\Delta + S_{\text{opt}}) \right] \Delta
\]

\[
= \left[ \frac{1}{2} (\Delta + \frac{1}{2}) \right] \Delta
\]

\[
Y_{\text{opt, real}} \text{ and } P_{\text{opt}} \text{ depend sensitively on } \gamma
\]

\[
\text{Pout depends sensitively on } \gamma
\]
MS: YAG rod, side pumped @ 807 nm by fiber-coupled diode laser.

\[ P = 370 \text{ W} \text{ with } \eta = 340 \text{ W} = \text{critical pump power absorbed} \]

\[
\eta = \frac{\eta_0}{\eta_0 + \eta_e} \approx 1.11 \text{ nm}
\]

\[ \eta = 15\% \]

\[ \eta_e = 3.8\% \]

Find output power and slope efficiency:

\[ \eta = \frac{\eta_0 - \eta_e}{\eta_0} \approx \eta_0 \]

\[ \frac{\eta_0}{\eta_0 + \eta_e} \approx 1.11 \]

\[
\frac{P}{P_0} = 1 - \frac{\eta_0}{\eta_0 + \eta_e} \approx 0.081 + 0.038 = 0.12
\]

\[ P = \frac{P_0}{\eta_0} \text{ for uniform temperature gradient} \]

\[ P = \frac{P_0}{\eta_0} \text{ in reality} \]

\[ P = \frac{P_0}{\eta_0} \text{ in reality} \]

\[ P = \frac{P_0}{\eta_0} \text{ in reality} \]

\[ P = \left( \frac{0.80}{0.81} \right) \left( \frac{62.9}{62.9} \right) = 0.704 \text{ cm}^3 \]

\[ V = 2.1 \cdot 10^{-15} \text{ J} \]

\[ V = 1.5 \cdot 10^{-15} \text{ V} \]
\[ P_{out} = A_b I_5 \frac{\chi}{h} \left( \frac{h \nu}{h \nu_h} - 1 \right) \]

\[ I_5 \approx \frac{h \nu = 1.17 eV}{1.87 \times 10^{19}} = 2.90 \times 10^{16} \]

\[ I_5 = 2.90 \times 10^{16} \text{ W/cm}^2 \]

\[ A_b \Rightarrow \{ \text{FWHM} = \sqrt{2} \sigma \text{, } \omega_c = 1.64 \text{ mm}, \quad A = \pi \left( \frac{\text{FWHM}}{2} \right) = 0.021 \text{ cm}^2 \} \]

Recall we found

\[ P = \frac{\pi \omega_c^2}{2} I_5 \]

\[ A_b = \frac{\pi \omega_c^2}{2} = 0.021 \text{ cm}^2 \]

\[ x_5 A_b = 89 W \]

\[ P_{out} = (89 W)(0.081) \left( \frac{740 W}{63 W} - 1 \right) = 72 W \sqrt{\text{W}} \]

\[ \eta = A_b I_5 \frac{\chi}{h} \frac{1}{h \nu_h} = 0.11 = 11\% \sqrt{\text{W}} \]

From Sect 36 E2

\[ P_{ax} = 62 W \]

\[ \eta \approx 25\% \quad (I \text{ think}) \]

Claim to be the highest reported TEM_{00} mode output for a single laser seed,

\[ [\text{Gally et al.}; \text{opt lett } 21, 210 (1990)] \]