1) Maximum Likelihood Method:  
Given the following probability distribution function for the variable x and constant \( \lambda \),  
\[ p(x, \lambda) = \lambda e^{-x\lambda} \]  
and three measurements of x (\( x_1 = 3 \) sec, \( x_2 = 5 \) sec, \( x_3 = 4 \) sec):  
a) Write down the likelihood function for this problem.  
b) Use the Maximum Likelihood Method to calculate \( \lambda \).

2) *Barlow 6.2*  
A trolley moves along a track with a constant speed, which you need to measure. It passes through the point \( d = 0 \) at exactly \( t = 0 \). At certain fixed distances, determined by sensing devices on the track, the time is measured with an error of 0.1 sec. The results are:  

<table>
<thead>
<tr>
<th>Time t (seconds)</th>
<th>1.1</th>
<th>2.2</th>
<th>2.9</th>
<th>4.1</th>
<th>5.0</th>
<th>5.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance d (mm)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

Find the velocity and the \( \chi^2 \).

3) *Barlow 4.5*  
If a current is determined by measuring a voltage of \( 45 \pm 1 \) V through a resistance of \( 900 \pm 10 \) \( \Omega \), what is its value and error?

4) We wish to determine the index of refraction (n) of a medium by measuring how much light bends when it travels from one medium to another. The law relating angles and indexes of refraction is Snell's Law:  
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2. \]  
Given the following measurements find \( n_2 \) and using propagation of errors the uncertainty of \( n_2 \).  
\( n_1 = 1.0000 \)  
\( \theta_1 = 22.03 \pm 0.2 \) degrees  
\( \theta_2 = 14.45 \pm 0.2 \) degrees

5) We wish to determine the acceleration due to gravity (g).  
a) Given the following data and the relationship:  
\[ x = 0.5gt^2 \]  
use the least squares technique to find the best value of \( g \). Assume the error in each x measurement is 0.01 m and the time is measured exactly.  

<table>
<thead>
<tr>
<th>( x ) (m)</th>
<th>( t ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>0.44</td>
<td>0.3</td>
</tr>
<tr>
<td>1.23</td>
<td>0.5</td>
</tr>
<tr>
<td>2.40</td>
<td>0.7</td>
</tr>
</tbody>
</table>

b) What is the value of the chi-square for this problem?  
c) How many degrees of freedom are there in this problem?  
d) Estimate the probability to get a chi-square per degree of freedom larger than what you obtain using your results from parts b) and c).
6) Two different experiments have measured the mass of the X boson. Experiment #1 measured 1.00 ± 0.01 gm while experiment 2 measured 1.04 ± 0.02 gm. What is the best estimate of the mass of the X boson if we combine the two experiments?

Calculate the $\chi^2$ for the two measurements in this problem using:

$$\chi^2 = \sum_{i=1}^{2} \frac{(m_i - m)^2}{\sigma_i^2}$$

with $m_i$ the measurement from experiment $i$ and $\sigma_i$ the standard deviation of the measurement, and $m$ the best estimate of the mass obtained by combining the two experiments. How many degrees of freedom are there in this problem? What's the probability of getting a value of $\chi^2$ greater than or equal to the one above?