Bessel’s Correction

The Root Mean Square estimator for $\sigma^2$ of an unknown mean $\mu$ is naively given as:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2 \quad \text{where} \quad \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Bessel showed that this estimator is biased (skewed), not surprising since it depends on $x_i^2$. Bessel’s corrected formula is given below (for proof see Barlow).

$$\sigma^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (x_i - \bar{x})^2 \quad \text{where} \quad \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Show in the Monte Carlo generated plot below are averaged $\sigma^2$ of 1 million trials each of $\sigma^2$’s estimated from N samples of a random gaussian distribution with $\mu=0$ and $\sigma^2=1$. Red points have Bessel’s correction while the blue points use the naïve formula. The curve superimposed on the blue points is $(N-1)/N$. 