Homework 7 Solutions

Ch. 28: #28
Ch. 29: #1, 9, 18, 21, 24, 37

28)

\[ E = 6.0 \text{ V} \]
\[ R_1 = 100 \Omega \]
\[ R_2 = R_3 = 50 \Omega \]
\[ R_4 = 75 \Omega \]

a)

resistors R2, R3 and R4 are in parallel, so their equivalent resistance, \( R_{234} \), is:

\[ R_{234} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{R_2 R_3 R_4}{R_2 + R_3 + R_4} \]

\( R_{234} \) and R1 are in series, so their equivalent resistance, \( R_{eq} \), is:

\[ R_{eq} = R_1 + R_{234} = R_1 + \frac{R_2 R_3 R_4}{R_2 R_3 + R_2 R_4 + R_3 R_4} \]

plugging in the given values for resistance yields:

\[ R_{eq} = 100 \Omega + \frac{(50 \Omega)(50 \Omega)(75 \Omega)}{(50 \Omega)(50 \Omega) + (50 \Omega)(75 \Omega) + (50 \Omega)(75 \Omega)} = \]

\[ 100 \Omega + \frac{1.875 \times 10^5 \Omega^3}{2.5 \times 10^3 \Omega^2 + 3.75 \times 10^3 \Omega^2 + 3.75 \times 10^3 \Omega^2} = \]

\[ 100 \Omega + \frac{1.875 \times 10^5 \Omega^3}{10^4 \Omega^2} = 100 \Omega + 1.875 \times 10^2 \Omega = 118.75 \Omega \]

= 120 \Omega

(2 significant figures)
b) using Kirchoff's voltage law,
\[ \varepsilon - i_1 R_1 - i_2 R_2 = 0 \]

the current out of the battery is equal to the current into \( R_1 \)
\[ i_1 = \frac{\varepsilon}{R_{eq}} = \frac{6.0 \text{ V}}{120 \Omega} = 5.0 \times 10^{-2} \text{ A} \]

solving the loop equation from above for \( i_2 \) yields:
\[ i_2 = \frac{\varepsilon - i_1 R_1}{R_2} = \frac{6.0 \text{ V} - (5.0 \times 10^{-2} \text{ A}) (100 \Omega)}{50 \Omega} = \frac{6.0 \text{ V} - 5.0 \text{ V}}{50 \Omega} = \frac{1.0 \text{ V}}{50 \Omega} = 2.0 \times 10^{-2} \text{ A} \]

using Kirchoff's voltage law on a loop with just \( R_2 \) and \( R_4 \) (and choosing \( i_4 \) to point away from \( R_3 \)) gives:
\[ i_2 R_2 = i_4 R_4 = 0 \]

solving for \( i_4 \) yields:
\[ i_4 = \frac{i_2 R_2}{R_4} = \frac{(2.0 \times 10^{-2} \text{ A}) (50 \Omega)}{(75 \Omega)} = 1.3 \times 10^{-2} \text{ A} \]

similarly for \( R_3 \) and \( R_4 \),
\[ i_3 R_3 = i_4 R_4 = 0 \text{ and} \]
\[ i_3 = \frac{i_4 R_4}{R_3} = \frac{(1.3 \times 10^{-2} \text{ A}) (75 \Omega)}{(50 \Omega)} = 2.0 \times 10^{-2} \text{ A} \]

you could also use Kirchoff's current law
\[ i_1 = i_2 + i_3 + i_4 \]
1) 

\[ \| \vec{v} \| = 550 \frac{m}{s} \]
\[ \| \vec{B} \| = 0.045 T \]
\[ q_a = 3.2 \times 10^{-9} \text{ C} \]
\[ m_a = 6.6 \times 10^{-27} \text{ kg} \]
\[ \phi = 52^\circ \]

a) 

The magnitude of the force on a moving charged particle due to a magnetic field is given by
\[ \| \vec{F}_B \| = \frac{q}{\| \vec{v} \| \| \vec{B} \| \sin(\phi)} \]
\[ = (3.2 \times 10^{-19} \text{ C}) \left( 5.50 \times 10^{-2} \frac{m}{s} \right) (4.5 \times 10^{-2} \text{ T}) \sin(52^\circ) \]
\[ = 6.2 \times 10^{-18} \text{ N} \]

b) 

Newton's second law
\[ \vec{F} = m \vec{a} \]

Thus, the acceleration of the particle due to the magnetic field is
\[ \| \vec{a} \| = \frac{\| \vec{F}_B \|}{m_a} = \frac{6.2 \times 10^{-18} \text{ N}}{6.6 \times 10^{-27} \text{ kg}} = 9.5 \times 10^8 \frac{m}{s^2} \]

c) 

Since \[ \vec{F}_B \perp \vec{v} \], it does no work on the particle (\[ \vec{W} = \vec{F} \cdot \vec{a} = 0 \] if \( \vec{F} \) and \( \vec{a} \) are perpendicular)

By the work-energy theorem, if the work done is zero, the change in kinetic energy is zero; if the change in kinetic energy is zero, the change in the magnitude of the velocity is zero since the mass stays the same
\[ q = -e = -1.6 \times 10^{-19} \text{ C} \]
\[ m_e = 9.1 \times 10^{-31} \text{ kg} \]
\[ \Delta V_{\text{external}} = 1.0 \times 10^3 \text{ V} \]
\[ d = 2.0 \times 10^{-2} \text{ m} \]
\[ \Delta V_{\text{plates}} = 1.0 \times 10^2 \text{ V} \]

\section*{a)}

by Newton's first law, for the velocity to remain constant, the net force on the electron must be zero

\[ \mathbf{F}_{\text{net}} = 0 \]

\[ \mathbf{F}_{\text{net}} = \mathbf{F}_B + \mathbf{F}_E = |q| v B \sin \phi + q \mathbf{E} \]

the magnetic field is perpendicular to the direction the electron is traveling and hence its velocity, so \( \phi = \frac{\pi}{2} \) and \( \sin \frac{\pi}{2} = 1 \)

the electric field between two parallel plates is given by

\[ \mathbf{E} = \frac{\Delta V_{\text{plates}}}{d} \] (Eq. 25 - 42 of text)

by energy conservation,

\[ U_i + T_i = U_f + T_f \]

(where \( U \) is the potential energy of an object and \( T \) the kinetic energy)

before moving through the potential the electron has an initial kinetic energy, \( T_i = 0 \)

\[ U_f - U_i = -T_f \]

putting in the definition of \( T_f \)

\[ -\Delta U = \frac{1}{2} m v_f^2 \]

\[ \Delta U = q \Delta V \]

\[ \Rightarrow v_f = \sqrt{-\frac{2 q \Delta V}{m}} \]

putting in our values for \( q, m, \Delta V \) and calling \( v_f, v \)

\[ v = \sqrt{\frac{2 e \Delta V_{\text{external}}}{m_e}} \]

putting this into the equation for \( \mathbf{F}_{\text{net}} \) and setting \( \mathbf{F}_{\text{net}} = 0 \), we get

\[ 0 = e \sqrt{\frac{2 e \Delta V_{\text{external}}}{m_e}} B - e \frac{\Delta V_{\text{plates}}}{d} \]
solving for $B$

$$B = \frac{\Delta V_{\text{plates}}}{d} \sqrt{\frac{m_e}{2 e \Delta V_{\text{external}}}}$$

checking the units of this answer to make sure they are units of magnetic field ($T$)

$$V \sqrt{\frac{\text{kg}}{\text{C m}}} = \frac{\frac{\text{C}}{\text{m}}}{\frac{\text{C}}{\text{m}}} = \frac{\text{N m}}{\text{C m}} \sqrt{\frac{\text{kg}}{\text{N m}}}$$

$$= \frac{N}{C} \sqrt{\frac{\text{kg}}{s^2 \text{m}}} = \frac{N}{C} \sqrt{\frac{s^2}{\text{m}^2}} = \frac{N}{C \frac{m}{s}} = T$$

plugging in our numbers

$$B = \frac{(1.0 \times 10^2 \text{ V})}{(2.0 \times 10^{-2} \text{ m})} \sqrt{\frac{(9.1 \times 10^{-31} \text{ kg})}{2 (1.6 \times 10^{-19} \text{ kg}) (1.0 \times 10^3 \text{ V})}}$$

$$= 2.7 \times 10^{-4} \text{ T}$$

18) 

$$B = 4.50 \times 10^{-2} \text{ T}$$

$$q = e \text{ (singly charged ion positive ion)}$$

$$= 1.60 \times 10^{-19} \text{ C}$$

number of revolutions $= 7.00$

t$_7 \text{ revolutions} = 1.9 \times 10^{-3} \text{ s}$

frequency is defined as one revolution per unit time so we divide the number of revolutions by the time it took to make those

$$f = \frac{7.00}{1.9 \times 10^{-3} \text{ s}} = 5.43 \times 10^3 \text{ Hz}$$

using Eq. 29 - 18 from the text for frequency of oscillation in a magnetic field

$$f = \frac{qB}{2 \pi m}$$

and solving for $m$ yields:

$$m = \frac{qB}{2 \pi f}$$

$$= \frac{(1.60 \times 10^{-19} \text{ C}) (4.50 \times 10^{-2} \text{ T})}{2 \pi (5.43 \times 10^3 \text{ Hz})} = 2.11 \times 10^{-25} \text{ kg}$$

converting to atomic mass units ($1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$)

$$m = 1.27 \times 10^{-27} \text{ u}$$
### 21)

we want the electrons to travel on a circular path of \( r \leq d \)

for charged particles traveling perpendicular to a magnetic field, the radius of curvature for their path is given by

\[
r = \frac{m v}{|q| B}
\]

(Eq. 29 - 16 of text)

the kinetic energy of a particle is given by

\[
K = \frac{1}{2} m v^2
\]

solving for \( v \),

\[
v = \sqrt{\frac{2 K}{m}}
\]

putting the equation for \( v \) into the equation

for \( r \) and the equation for \( r \) into the condition on \( r \) gives

\[
\frac{m}{|q| B} \sqrt{\frac{2 K}{m}} \leq d
\]

solving for \( B \) and putting in \( |q| = |e| = e \)

\[
B \leq \sqrt{\frac{2 m^2 K}{e^2 m d^2}} = \sqrt{\frac{2 m e K}{e^2 d^2}}
\]

this equation holds true only for \( B \) pointing out of the page because then the magnetic force points toward the center of the circular path

### 24)

\[
T_p = T_d = T_\alpha = T \quad (\text{kinetic energy})
\]

\( q_p = e; \quad q_d = e; \quad q_\alpha = 2 \, e \quad (\text{charge}) \)

\( m_p = u; \quad m_d = 2 \, u; \quad m_\alpha = 4 \, u \quad (\text{mass}) \)

\[ \phi = \frac{\pi}{2} \]

as in the previous problem

\[
r = \frac{m v}{q B} = \frac{m}{q B} \sqrt{\frac{2 T}{m}} = \frac{1}{q B} \sqrt{2 m T}
\]

\( T \) and \( B \) are the same for each particle
\[ r_p = \frac{1}{eB} \sqrt{2} uT; \]
\[ r_d = \frac{1}{eB} \sqrt{2 (2u) T} = \sqrt{2} \frac{1}{eB} \sqrt{2} uT = \sqrt{2} r_p \]
\[ r_a = \frac{1}{2eB} \sqrt{2 (4u) T} = \frac{\sqrt{2}}{2} \frac{1}{eB} \sqrt{2} uT = \frac{2}{2} r_p = r_p \]

Since \( \sqrt{2} > 1 \),
\[ r_d > r_p = r_a \]

\[ \square 37) \]

\[ m = 1.0 \text{ kg} \]
\[ L = 1.0 \text{ m} \]
\[ i = 50 \text{ A} \]
\[ \mu_s = 0.60 \]

The force of static friction, \( f_s^x \), against the force generated by the magnetic field and the current will be in the same plane as the magnetic field, \( \vec{B} \), and the force generated by the magnetic field \( \vec{F}_B = i \vec{L} \times \vec{B} = iLB \) for \( \vec{B} \perp \vec{L} \). \( \vec{F}_B \) makes an angle \( \theta \) with \( f_s^x \).

Since \( f_s^x \perp \vec{N} \), the normal vector, and \( \vec{B} \perp \vec{F}_B \), then the angle between \( \vec{B} \) and \( \vec{N} \) is also \( \theta \). In order for the rod to move their must be just above zero acceleration and therefore the net force must be just above zero.

We'll call the direction of the static friction force \( x \) and the direction of the normal force \( z \)

\[ F_{net_x} = iLB \cos \theta - \mu_s N \]
\[ F_{net_z} = iLB \sin \theta + N - mg \]

Since there is no acceleration upwards or downwards
\[ F_{net_z} = 0 \Rightarrow N = mg - iLB \sin \theta \]

Plugging this value for \( N \) into \( F_{net_x} \) and setting \( F_{net_x} = 0 \) yields
\[ iLB \cos \theta - \mu_s (mg - iLB \sin \theta) = 0 \]
Solving for \( B \) gives
\[ B = \frac{\mu_s mg}{iL (\cos \theta + \mu_s \sin \theta)} \]

The minimal value of \( B \) will occur when \( \cos \theta + \mu_s \sin \theta \) is maximized.

To find the maximum of \( \cos \theta + \mu_s \sin \theta \), take
\[ \frac{d}{d\theta} (\cos \theta + \mu_s \sin \theta) = 0 \]
\[ -\sin \theta + \mu_s \cos \theta = 0 \]
Solving for \( \theta \) gives
\[ \theta = \arctan (\mu_s) \] [also denoted : \( \tan^{-1} (\mu_s) \)]
putting in our value for $\mu_s$ gives
$\theta = 0.55 \text{ radians or } 31^\circ$

then,

$$B = \frac{(0.60) (1.0 \text{ kg}) (9.8 \frac{\text{m}}{\text{s}^2})}{(50 \text{ A}) (1.0 \text{ m}) (\cos 31^\circ + (0.60) \sin 31^\circ)} = 0.10 \text{ T}$$
Without the voltmeter current: \( V = iR \)

With the voltmeter current we replace \( R \) with the equivalent resistance \( R' \) then: \( V' = i'R' \)

Since the equivalent resistance is just \( R_v \) and \( R \) in parallel then

\[
\frac{1}{R'} = \frac{1}{R} + \frac{1}{R_v} \quad \text{or rearranging} \quad \frac{1}{R} = \frac{1}{R'} - \frac{1}{R_v}
\]