I-1 2.88.B
(a) The tangerine will slow down as it rises, like all thrown objects. So it passes 3 at the highest speed, followed by 2, and 1 at the lowest speed.

(b) The windows are of equal size, so the tangerine requires less time for windows that it passes at high speed. The longest time is window 1, then 2, and 3 takes the least time.

(c) Acceleration is constant at $-9.8\,\text{m/s}^2$ for all projectiles, so $a$ is the same at each window.

(d) Change in speed is related to acceleration by $a = \frac{\Delta v}{\Delta t}$, or $\Delta v = a \cdot \Delta t$. "$a$" is the same for each window, so the largest $\Delta v$ will occur at the windows where the longest time passes. Thus, $\Delta v$ is largest for 1, medium for 2, and smallest for 3.

II-2 2.99.c
The question prompts us to look at equation 2-16:
$$\frac{v^2}{2} = v_0^2 + 2a\Delta x$$
In both cases, $\Delta x$ is the same: the height of the cliff. "$a$" is $-9.8\,\text{m/s}^2$; the only change between the two cases is $v_0$. It's positive the first time, negative the second time, but the speed, $s = |v_1|$, is the same. In the first case, then, we have $(v_0)^2$ and in the second, $(-v_0)^2$. Because the velocity is being squared, the minus sign has no effect. $v_2$ will be the same in both cases.

II-3 2.97.c
Our spaceship has these properties: $a = 9.8\,\text{m/s}^2$, with $+$ being "forward." For the ship,

$$V_0 = 0, \quad V_f = \left(\frac{1}{10}\right)(3(10)^7\text{m/s}) = 3(10)^6\text{m/s}$$

(a) $V_f = V_0 + at_f$  \hspace{2cm} \text{OR} \hspace{2cm} t_f = \frac{V_f - V_0}{a}$

\[ a = \frac{\Delta V}{\Delta t} \rightarrow \Delta t = \frac{\Delta V}{a} \]

\[ t_f = \Delta t = \frac{3(10)^6\text{m/s}}{9.8\,\text{m/s}^2} = \frac{3(10)^6}{9.8}\quad \text{5 days} \]

(b) There are two easy ways to do this part:

Using (a)

$$\Delta x = V_t + \frac{1}{2}at^2$$

\[ \Delta x = 0 + \frac{1}{2}(9.8\,\text{m/s}^2)(3.05(10)^6)^2 \]

\[ \approx 4.59\,(10)^{13}\,\text{m} \]

~8 times further than Pluto.

Independant of (a)

$$V^2 = V_0^2 + 2a\Delta x$$

$$\Delta x = \frac{V_f^2 - V_0^2}{2a} = \frac{(3(10)^6\text{m/s})^2 - 0}{2(9.8\,\text{m/s}^2)}$$

\[ = 4.59(10)^{13}\,\text{m} \]
Since the acceleration is constant, we can use just about any of the formulas from Chapter 2.

a) For a constant acceleration, equation (2-17) from the book looks good: \( x - x_0 = \frac{1}{2} (v_0 + v) t \). We know the difference in distance between the first and second point is 60.0 m = \( x - x_0 \). (So we are saying \( x_0 \) = first point, \( x \) = second point). Similarly for velocity, \( v = 15 \text{ m/s} \), the velocity at point 2, and \( v_0 \) = velocity at point 1, which we want to find. \( t \) = time elapsed between \( x \) and \( x_0 \), in this case \( t = 6.00 \text{ sec} \).

So \( 60.0 \text{ m} = \frac{1}{2} (15 + v_0) (6.00) \Rightarrow 2(10.0) = 15 + v_0 \)

\[ v_0 = 5 \text{ m/s} \]

b) We can use \( v = v_0 + at \) to find \( a \), since we know the velocities at both points and the time elapsed between the points. Same values for \( v \) & \( t \) as part a) \( v = 15 \text{ m/s} \) \( t = 6.00 \text{ sec} \), \( v_0 = 5 \text{ m/s} \) (from part a)

So \( 15 = 5 + a(6.00) \Rightarrow 10 = 6a \)

\[ a = 1.67 \text{ m/s}^2 \] it is positive since the car accelerated from 5 m/s to 15 m/s.

c) Here we can consider the car starting from rest, \( v_0 = 0 \), with an acceleration \( a = 1.67 \text{ m/s}^2 \). When the velocity reaches \( v = 5 \text{ m/s} \) (the velocity at point 1) we want to know the distance traveled.

So there is a relation if we have initial + final velocity, acceleration, and distance: \( v^2 = v_0^2 + 2a(x-x_0) \) In our case, \( v = v_i = 5 \text{ m/s} \), \( v_0 = 0 \) (starts from rest), \( a = 1.67 \text{ m/s}^2 \), and \( x-x_0 \) is what we want to find. We could say that \( x_0 = 0 \), and then solve for \( x \), the final position of the particle in this relation, which in our case would be the position at point 1. So we have \( v^2 = 2ax \) and we want to solve for \( x \).

\[ x = \frac{(5 \text{ m/s})^2}{2 (1.67 \text{ m/s}^2)} = 7.50 \text{ m} \]
d) The car starts at (0,0), moves under a constant acceleration $a = 1.67 \text{ m/s}^2$, attains a velocity of $5 \text{ m/s}$ when it is 7.5 m down the road (point 1). To make our graphs, we need to know how long this takes to reach the first point. We can use $v = v_0 + at$ to find the time from start to point 1: 

$$5 \text{ m/s} = 0 \text{ m/s} + (1.67 \text{ m/s}^2) t \implies t = 3 \text{ sec}.$$ 

So it takes 3 sec to reach point 1, and another 6 sec after that to reach point 2 ($t = 3 + 6 = 9 \text{ sec}$) and ($x = 7.5 \text{ m} + 60 \text{ m} = 67.5 \text{ m}$).

So our graphs should look something like:

**Distance vs. Time**

**Velocity vs. Time**
2.47B)

A) Assuming the armadillo starts at an initial velocity $V_0$ and then accelerates downward (according to gravity) as it moves upward:

$$a = g = -9.8 \text{ m/s}^2$$

$$\Delta x = V_0t + \frac{1}{2}at^2,$$

where $\Delta x = 0.544 \text{ m}$ and $t = 0.200 \text{ s}$

Solving algebraically for $V_0$ gives:

$$V_0 = \Delta x - \frac{1}{2}at^2 = 0.544 \text{ m} + (0.5)(9.8 \text{ m/s}^2)(0.200 \text{ s})$$

$$V_0 = 3.7 \text{ m/s}$$

B) The armadillo's velocity at $x = 0.544 \text{ m}$ is given by:

$$v = V_0 + at = 1.74 \text{ m/s} \quad V_0 = 3.7 \text{ m/s}, \quad a = -9.8 \text{ m/s}^2, \quad t = 0.25 \text{ s}$$

The speed is the magnitude of the velocity $|v|$

$$|v| = 1.74 \text{ m/s}$$

C) The armadillo keeps going upward until its velocity reaches zero.

$$v^2 = v_0^2 + 2a\Delta x,$$

$$\Delta x = \frac{v_0^2}{2a}, \quad V_0 = 0, \quad V_e = 3.7 \text{ m/s}, \quad a = -9.8 \text{ m/s}^2$$

$$\Delta x = \frac{(3.7)^2}{2(-9.8)} \approx 0.70 \text{ m}$$
2.56) 

3) Find the free-fall acceleration of the planet (system).
   - One-dimensional motion
   - Constant acceleration

Here: \[ \Delta x = v_0 \Delta t + \frac{1}{2} a(\Delta t)^2 \]

To find "free-fall" acceleration, look only at the "free-fall" portion of the curve. Free-fall begins at \( t = 2.5 \) s and ends at \( t = 5.0 \) s and the \( \Delta x = -25 \) m while \( v_0 = 0 \) m/s.

\[ a \text{ (m/s}^2) = \frac{2 \Delta x}{(\Delta t)^2} = \frac{2(-25 \text{ m})}{(5.0 \text{ s} - 2.5 \text{ s})^2} \]

\[ a \text{ (m/s}^2) = \frac{50 \text{ m}}{6.25 \text{ s}^2} = -8.0 \text{ m/s}^2 \]

(Units match)

b) Find the initial velocity of the ball.

There are several ways to solve for the initial velocity.

One way is to use \( \Delta x = v_0 \Delta t + \frac{1}{2} a(\Delta t)^2 \) over the interval \((0 \text{ s} \leq t \leq 5 \text{ s})\). Here, \( \Delta x = 0 \) m, \( \Delta t = 5.0 \) sec, \( a = -8.0 \text{ m/s}^2 \).

\[ \Delta x = 0 \text{ m} = v_0 \Delta t + \frac{1}{2} a(\Delta t)^2 \]

\[ v_0 \text{ (m/s)} = -\frac{1}{2} a \Delta t = -\frac{1}{2} (-8.0 \text{ m/s}^2)(5.0 \text{ s}) \]

\[ v_0 \text{ (m/s)} = 20 \text{ m/s} \quad \text{(Units match)} \]
It's relatively easy to find the position of a single falling diamond as a function of time. If we let the release point be at $y = 0$, then the first diamond obeys $y_1(t) = \frac{1}{2}at^2$, since it is released from rest ($v_0 = 0$).

What about the other diamond? How do we deal with the 1s delay? We can't say that $y_2(t) = \frac{1}{2}at^2$ also because that would place both diamonds at the same position. When the first diamond has fallen for one second, the second diamond is yet to be released, and needs to have $x(1s) = 0$. The trick is to write it this way: $y_2(t) = \frac{1}{2}a(t-1s)^2$. Now the equations "know" that diamond #2 has fallen for a second less.

The question asks when the two will be 10m apart. #1 will be lower, so we want $y_2(t) - y_1(t) = 10 \text{ m}$. So, with $a = -9.8 \text{ m/s}^2$...

\[
y = 0 \quad 10\text{m} = y_2(t) - y_1(t) = \frac{1}{2}a(t-1s)^2 - \frac{1}{2}at^2
\]

\[
= -4.9\text{ m/s}^2(t^2 - 2st + 1s^2) + 4.9\text{ m/s}^2t^2
\]

\[
10\text{m} = 9.8\text{m/s} t = 4.9\text{ m}
\]

\[
14.9\text{m} = 9.8\text{m/s} t \quad \rightarrow \quad t = 1.525\text{ s}
\]
HW2 Group II Solutions

3-Q10, C

(a) Keep in mind that in the dot-product definition, $\alpha \cos \phi$, the angle $\phi$ is measured when the vectors are placed tail-to-tail:

So the proper angle for vectors $\mathbf{B}$ and $\mathbf{E}$ is actually $180^\circ - \Theta = \phi_{AB}$, while for $\mathbf{B}$ and $\mathbf{E}$ you can just use $\Theta = \phi_{AE} = \phi_{AB}$. Since all four vectors have the same magnitude, the only thing which can change their dot product with $\mathbf{A}$ is the angle. Any two with the same angle — $\mathbf{B}$ and $\mathbf{E}$, or $\mathbf{B}$ and $\mathbf{E}$ — will have the same dot product with $\mathbf{A}$.

(b) In $\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \phi$, $\mathbf{A}$ and $\mathbf{B}$ (written without vectors) are magnitudes, so they are always positive. To get a negative product, then, $\cos \phi$ must be negative.

$\phi$ can't be more than $180^\circ$, because we always choose the smaller angle.

So, for what values of $\phi$ can cosine be negative? From $90^\circ$ to $270^\circ$, or the 2nd and 3rd quadrants. Only $90^\circ$ to $180^\circ$ is relevant for vector products. So, the conclusion is that any dot product where $\phi$ is more than $90^\circ$ will be negative. $\mathbf{B}$ and $\mathbf{E}$ fit that description in our problem.

3-1, C

The idea here is just to see that you need to know more than just magnitudes to find a resultant vector. The angles between vectors matter; they change things.

(a) Like them up:

\[ \begin{align*}
4m & \rightarrow 3m \\
7m & \rightarrow \\
\end{align*} \]

(b) Pointing opposite:

\[ \begin{align*}
4m & \rightarrow 3m \\
\rightarrow & \rightarrow \\
\end{align*} \]

(c) At a right angle:

\[ \begin{align*}
4m & \rightarrow \text{Length: It's a hypotenuse of a triangle with legs} \\
4m \text{ and } 3m: \\
\sqrt{4^2 + 3^2} &= 5
\end{align*} \]
Problem 2 Chapter 3:

Note that this is nothing but adding three known vectors in geometrical method.

In order to find the final displacement for Bank robbers, we have to add three given displacement. We use the **head to tail** method:

1. Start from the bank draw a straight line in 45 degree to south of east and considering the map scale (upper left corner of map) we continue for 32 KM to point “A”

2. Draw a 53 Km long straight line from point “A” in 26 degree north of west to point “B”.

3. Draw the third vector from “B” towards 18 degree east of south for 26 Km.

The sum of these three vectors is the straight lone from start (Bank) to finish (Walpole).
\(\begin{align*}
\text{II-11} \\
3-5, c
\end{align*}\)

(a) To find a vector's magnitude from components, square each component, add them, and take a square root.

\[ |\vec{A}| = \sqrt{(-25\text{m})^2 + (40\text{m})^2} = \sqrt{625 + 1600} = \sqrt{225} = 15\text{ m} \]

(b) To get the angle from +x by using components, use a tangent:

\[ \tan \Theta = \frac{Ax}{Ay} = \frac{40\text{m}}{-25\text{m}} \quad \Rightarrow \quad \Theta = -58^\circ \]

But, as always, be careful about that angle! Always check that it's in the correct quadrant. This time it's not. Tangent is negative in quadrants 2 and 4. To move this angle from 4th to 2nd, add 180°:

\[ \Theta = 122^\circ \]

An alternate method is to find the angle inside the triangle.

\[ \tan \phi = \frac{40\text{m}}{25\text{m}} \quad \Rightarrow \quad \phi = 58^\circ \]

\(\Theta\) is the angle from +x to our vector, so it must be 180° - 58° = 122°.

This method avoids confusion over which quadrant we're in.

\(\begin{align*}
\text{II-12} \\
3-7, b
\end{align*}\)

The wheel has rolled half a revolution, which means it's moved half a circumference to the side.

\(\begin{align*}
\text{Call the displacement vector } \vec{D}. \\
\text{Then } D_x = x = 1.4\text{ m}, \quad D_y = 2r = 0.9\text{ m}
\end{align*}\)

(a) \[ D = \sqrt{(1.4\text{m})^2 + (0.9\text{m})^2} = 1.67\text{ m} \]

(b) \[ \Theta = \arctan \left( \frac{0.9}{1.4} \right) = 32.7^\circ \]

\(\begin{align*}
\text{II-13} \\
3-17, c
\end{align*}\)

\[\vec{a} = 4\hat{i} - 3\hat{j} + 1\hat{k} \]

\[\vec{b} = -1\hat{i} + 1\hat{j} + 4\hat{k} \]

(a) \[\vec{a} + \vec{b} = (4 - 1)\hat{i} + (-3 + 1)\hat{j} + (1 + 4)\hat{k} = 3\hat{i} - 2\hat{j} + 5\hat{k} \]

(b) \[\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \]

\[ -\vec{b} = -1\hat{i} - 1\hat{j} - 4\hat{k} \]

\[\vec{a} + (-\vec{b}) = (4 + 1)\hat{i} + (-3 - 1)\hat{j} + (1 - 4)\hat{k} = 5\hat{i} - 4\hat{j} - 3\hat{k} \]

(c) Let \[\vec{z} = \vec{b} - \vec{a} \]

Then \[\vec{a} - \vec{b} - \vec{z} = 0. \]

If \[\vec{a} - \vec{b} + \vec{z} = 0, \]

then \[\vec{z} = -\vec{a} = -5\hat{i} + 4\hat{j} + 3\hat{k} \]
Assume two vectors \( \vec{A} \) & \( \vec{B} \).

Sum of them is \( \vec{A} + \vec{B} \),
Difference is \( \vec{A} - \vec{B} \).

When two vectors are perpendicular, their dot product would be zero. (\( \therefore \vec{A} \cdot \vec{B} = AB \cos 90^\circ = 0 \))

Try the dot product for the sum and the difference
\((\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B})\)
\[= \vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - \vec{B} \cdot \vec{B}\]
\[\vec{A} \cdot \vec{B} = AB \cos \phi = \vec{B} \cdot \vec{A}\]
\[\vec{A} \cdot \vec{A} = |A|^2 \quad \vec{B} \cdot \vec{B} = |B|^2\]

\(\therefore (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = |A|^2 - |B|^2\)

\(\Rightarrow\) The dot product would be zero only when
\(|A| = |B|\)
a) Show that $\vec{a} \cdot (\vec{b} \times \vec{a}) = 0$ for all vectors $\vec{a}$ and $\vec{b}$.

Let's do this for the general case where $\vec{a}$ and $\vec{b}$ can be any vector such that

$$\vec{a} = a_x \hat{x} + a_y \hat{y} + a_z \hat{z} \quad \text{and} \quad \vec{b} = b_x \hat{x} + b_y \hat{y} + b_z \hat{z}$$

where $a_x$, $a_y$, $a_z$, $b_x$, $b_y$ and $b_z$ can be any numbers and $\hat{x}, \hat{y}, \hat{z}$ correspond to $\hat{\imath}, \hat{j}, \hat{k}$ respectively.

Let's take $(\vec{b} \times \vec{a})$ first

$$(\vec{b} \times \vec{a}) = (b_y a_z - b_z a_y) \hat{x} + (b_z a_x - b_x a_z) \hat{y} + (b_x a_y - b_y a_x) \hat{z}$$

Now let's take $\vec{a} \cdot (\vec{b} \times \vec{a})$

$$\vec{a} \cdot (\vec{b} \times \vec{a}) = (b_y a_z - b_z a_y) a_x + (b_z a_x - b_x a_z) a_y + (b_x a_y - b_y a_x) a_z$$

$$= b_y a_z a_x - b_z a_y a_x + b_z a_x a_y - b_x a_z a_y + b_x a_y a_z - b_y a_x a_z$$

$$\uparrow \quad \text{cancel} \quad \uparrow \quad \text{cancel} \quad \uparrow \quad \text{cancel}$$

$$\Rightarrow \quad \vec{a} \cdot (\vec{b} \times \vec{a}) = 0 \quad \text{for any arbitrary vectors $\vec{a}$ and $\vec{b}$}$$

b) Given an angle $\theta$ between the vectors $\vec{a}$ and $\vec{b}$, what is the magnitude of the vector product $\vec{a} \times (\vec{b} \times \vec{a})$?

Let $\vec{c} = (\vec{b} \times \vec{a})$. $\vec{c}$ is a vector that is perpendicular to the plane that $\vec{a}$ and $\vec{b}$ lie in. For a vector product of two vectors $\vec{a}$ and $\vec{b}$, the two vectors $\vec{a}$ and $\vec{b}$ are in the same plane. For any two arbitrary
Vectors in space they will lie in the same plane. So that means that for $\vec{c} = (\vec{b} \times \vec{a})$ that $\vec{c}$ is perpendicular to both $\vec{b}$ and $\vec{a}$.

So the magnitude of $\vec{c}$ is

$$|\vec{c}| = |\vec{a}||\vec{b}| \sin \alpha$$

Let $\vec{d} = \vec{a} \times (\vec{b} \times \vec{a}) = \vec{a} \times \vec{c}$

The magnitude of $\vec{d}$ is

$$|\vec{d}| = |\vec{a}||\vec{c}| \sin \theta$$  where $\theta$ is the angle between $\vec{a}$ and $\vec{c}$.

We said that $\vec{c}$ is perpendicular to $\vec{a} \Rightarrow \theta = 90^\circ \Rightarrow \sin \theta = 1$

So

$$|\vec{d}| = |\vec{a}||\vec{c}| = (|\vec{a}|^2)(|\vec{b}|^2) \sin \theta = |\vec{d}|$$