1. (10 pts) Fill in the steps omitted in class to derive the one-loop electron self-energy correction in QED using dimensional regularization. That is, obtain Eq. (10.41) in Peskin and Schroeder with zero photon mass, $\mu = 0$, starting from

$$\Sigma_2(p) = -ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu (k' + m) \gamma_\mu}{[(p - k)^2 + i\epsilon][k^2 - m^2 + i\epsilon]}.$$ (1)

Assume that $p^2 < 0$.

2. Using dimensional regularization find the one-loop beta-function

$$\beta(\lambda) = \mu^2 \frac{d\lambda}{d\mu^2}$$

of the real scalar $\varphi^3$ theory with the Lagrangian density

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi_0 \partial^\mu \varphi_0 - \frac{m_0^2}{2} \varphi_0^2 - \frac{\lambda_0}{3!} \varphi_0^3$$ (2)

in six (!) space-time dimensions. Here $\varphi_0$ is the bare field, while $\lambda_0$ and $m_0$ are the bare coupling constant and the bare mass correspondingly.

a. (20 pts) Similar to how you did it in HW 3 for $\phi^4$ theory, rewrite the Lagrangian (2) in terms of renormalized physical fields $\varphi$, coupling $\lambda$, mass $m$ and the counterterms. By calculating the one-loop propagator and vertex corrections find the divergent ($\sim 1/\epsilon$ with $\epsilon = 6 - d$) parts of the coefficients of the counterterms $\delta_\lambda$ and $\delta_Z$. (Ignore the tadpole graph.) Note that we do not need the finite parts of the counterterms to find $\beta(\lambda)$!

b. (10 pts) Using the results of part a find $\beta(\lambda)$ following the steps of the QED beta-function calculation performed in class.