Particle Production in High Energy

Hadronic Collisions.

Collinear Factorization

When we considered DIS above, we have factorized EM & QCD parts of the diagram:

\[ F_2(x, Q^2) = \sum_f e_f^2 x q_f^T(x) \]

Thus, this is an example of collinear factorization.

In general, one writes:

\[ F_2(x, Q^2) = \sum_{f, \bar{f}, g} \int_0^1 d\xi \ C_2^f(\frac{x}{\xi}, Q^2, M^2) \rho_f^T(\frac{\xi}{\Lambda^2}) + o(\frac{m^2}{Q^2}) \]

\( C_2 \) coefficient for

\( \rho_f^T = \{ q_f^T, g_f^T, g \} \)

- Momentum fraction of the parton in \( p_f^T \)
\( \mu^2 \) is called factorization scale.

Note: \( C_2^f \) is perturbatively calculable, \( p^f \) is not (though one has DGLAP for \( p^f \)).

\( \Lambda^2 \lesssim \mu^2 \lesssim Q^2 \Rightarrow \) but \( F_2 \) does not depend on \( \mu^2 \), it is arbitrary \( \Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 (x, Q^2) = 0 \)

Write \( F_2 = (C_2^f \otimes p^f) \)

\( \Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = 0 = (\mu^2 \frac{d}{d\mu^2} C_2^f) \otimes p^f + C_2^f \otimes \mu^2 \frac{d}{d\mu^2} p^f \)

\( \Rightarrow \) what happens (separation of variables, \( C_2 \) depends on \( Q^2 \), only \( p^f \) depends on \( \Lambda^2 \)):

\[ \mu^2 \frac{d}{d\mu^2} p^f = \delta (d_3) \otimes p^f \sim \text{DGLAP evolution} \]

\[ \mu^2 \frac{d}{d\mu^2} C_2 = -\delta (d_3) \otimes C_2^f \]

\( \Rightarrow \mu^2 \frac{d}{d\mu^2} F_2 = -\delta \otimes C_2^f \otimes p^f + C_2^f \otimes \delta \otimes p^f = 0. \)

as desired.

\( \Rightarrow \) can "place" corrections into PDF or coefficient function.
Collinear factorization in DIS is a theorem which can be proven \( \Rightarrow \) must be right! (at large - \( \alpha_s \) only!)

\[ C_i^f = S \left( \frac{x}{\bar{x}} - 1 \right) e_i^2 \text{, } f = \text{quark only} \]

\[ \Rightarrow F_2(x, Q^2) = \sum_f \int_0^1 d\bar{x} \ S \left( \frac{x}{\bar{x}} - 1 \right) e_f^2 \ x \bar{x} f(x) \]

\[ = \sum_f e_f^2 \ x \bar{x} f(x) \text{ as expected!} \]

Jet Production in Hadronic Collisions.

Collinear factorization also applies to hadron-hadron collisions. Consider quark production:

A collision happens very fast on proton's time scale \( \Rightarrow \) factorization.