Homework Set No. 4, Physics 880.02

Deadline – Thursday, June 4, 2009

1. (20 pts) Similar to what we did in class, solve the DGLAP equation for gluon distribution

$$Q^2 \frac{\partial}{\partial Q^2} G(x, Q^2) = \frac{\alpha_s}{2\,\pi} \int_x^1 \frac{dx'}{x'} \, \gamma_{GG}(x/x') \, G(x', Q^2)$$

with

$$\gamma_{GG}(z) = \frac{2\,N_c}{z}$$

in the small-x asymptotics, but now with *fixed* coupling constant α_s (independent of Q^2). In particular show that, in the saddle point approximation, the small-x asymptotics for gluon distribution is given by

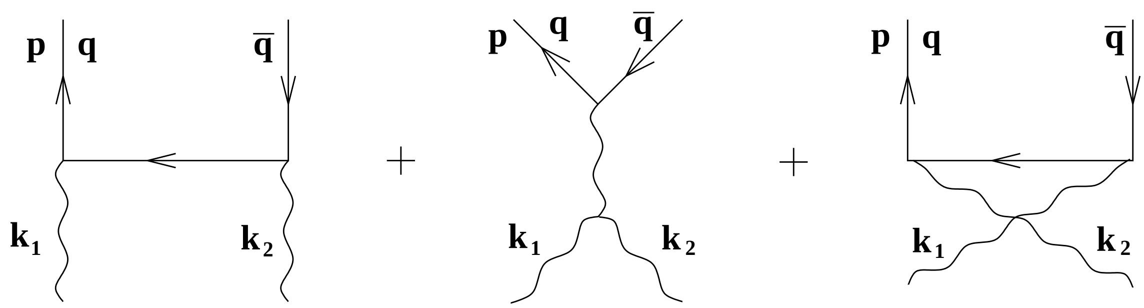
$$xG(x, Q^2) \sim \exp\left(2 \sqrt{\frac{\alpha_s\, N_c}{\pi} \, \ln \frac{1}{x} \, \ln \frac{Q^2}{Q_0^2}} \right).$$

This is called double logarithmic approximation (DLA) of DGLAP, since we resum both $\ln \frac{1}{x}$ and $\ln \frac{Q^2}{Q_0^2}$ in the new parameter $\alpha_s \, \ln \frac{1}{x} \, \ln \frac{Q^2}{Q_0^2}$.

2. (30 pts) Calculate the cross section for

gluon + gluon → quark + antiquark

at the Born level shown in the figure below. The figure is for the *amplitude*, which needs to be squared and multiplied by appropriate factors to get the cross section.



You should find

$$\mathcal{E}_p \frac{d\hat{\sigma}_{gg \to q\bar{q}}}{d^3 p} = \frac{3\,\alpha_s^2}{8\,\hat{s}} \, (\hat{t}^2 + \hat{u}^2) \left(\frac{4}{9\,\hat{t}\,\hat{u}} - \frac{1}{\hat{s}^2} \right) 2\,\delta(\hat{s} + \hat{t} + \hat{u}) \tag{1}$$

with the Mandelstam variables $\hat{s} = (k_1 + k_2)^2$, $\hat{t} = (k_1 - p)^2$, $\hat{u} = (k_2 - p)^2$. The factor of 2 in front of the δ-function in Eq. (1) comes from the fact that either the quark or the antiquark can carry momentum p. (q and $\bar{q}$ in the figure denote the quark and the antiquark. Time flows upward.) Assume that quarks are massless.

As a starting point you may take the formula derived in class:

$$\mathcal{E}_p \, \frac{d\hat{\sigma}_{gg\to q\bar{q}}}{d^3 p} \;=\; \frac{1}{16\,(2\,\pi)^2\,\mathcal{E}_1\,\mathcal{E}_2}\, 2\,\delta(\hat{s} + \hat{t} + \hat{u})\,\langle |M|^2 \rangle \tag{2}$$

where $\mathcal{E}_1 = k_1^0$, $\mathcal{E}_2 = k_2^0$, and M is the amplitude (sum of all three diagrams drawn above). Angle brackets $\langle \ldots \rangle$ denote summation over final state quantum numbers (polarizations and colors of the quarks) and averaging over initial state quantum numbers (polarizations and colors of the gluons). The factor of 2 mentioned above in Eq. (1) is already included in Eq. (2).

Hints:

After squaring the amplitude you may replace the polarization sums by

$$\sum_{\lambda=\pm 1} \epsilon_\mu^{*\,\lambda}(k)\,\epsilon_\nu^{\lambda}(k) \;\to\; -g_{\mu\nu}.$$

Also note that, as incoming gluons are physical,

$$k^\mu\,\epsilon_\mu^\lambda(k) \;=\; 0.$$

The following γ-matrix formulas may be useful:

$$\gamma^\mu\,\gamma^\nu\,\gamma_\mu \;=\; -2\,\gamma^\nu$$

$$\gamma^\mu\,\gamma^\nu\,\gamma^\rho\,\gamma_\mu \;=\; 4\,g^{\nu\rho}$$

$$\gamma^\mu\,\gamma^\nu\,\gamma^\rho\,\gamma^\sigma\,\gamma_\mu \;=\; -2\,\gamma^\sigma\,\gamma^\rho\,\gamma^\nu$$

$$\mathrm{tr}[\gamma^\mu\,\gamma^\nu] \;=\; 4\,g^{\mu\nu}$$

$$\mathrm{tr}[\gamma^\mu\,\gamma^\nu\,\gamma^\rho\,\gamma^\sigma] \;=\; 4\,(g^{\mu\nu}\,g^{\rho\sigma} + g^{\mu\sigma}\,g^{\nu\rho} - g^{\mu\rho}\,g^{\nu\sigma}).$$

For color traces the following expressions may come in handy:

$$T^a\,T^a \;=\; C_F\,\mathbf{1}$$

with

$$C_F \;=\; \frac{N_c^2 - 1}{2\,N_c},$$

$$\mathrm{tr}[T^a T^b T^a T^b] \;=\; -\frac{C_F}{2}$$

$$f^{abc}\,f^{abd} \;=\; N_c\,\delta^{cd}.$$