Yen Lee Loh
Research Statement

My present research focuses on the theory of strongly correlated electron systems. This is a branch of condensed matter physics that seeks to understand how the mutual interaction of a large number of electrons in a solid leads to interesting collective behavior. I have worked on a diverse range of topics which are all interconnected through the themes of magnetism, superconductivity, disorder, quantum criticality, and the Coulomb blockade. This statement describes my work since 2003.

1. Magnetic Droplets near Ferromagnetic Quantum Critical Points

2003–2006, with Dr. Vikram Tripathi and Dr. Misha Turlakov (Cavendish Laboratory)
Phys. Rev. B 71, 024429 (2005)

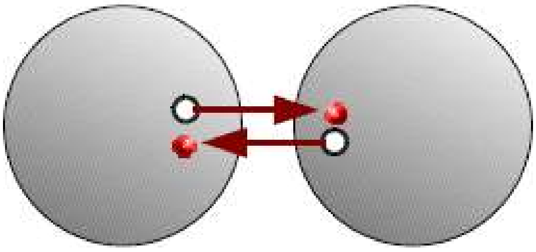
Metallic palladium and platinum have anomalously high magnetic susceptibilities because of their proximity to quantum phase transitions, so that 'giant' magnetic moments may form around impurities such as iron atoms, resulting in a magnetic susceptibility which varies sensitively with temperature, but sometimes shows deviations from the Curie law due to quantum fluctuations. Using various analytical and numerical techniques as well as drawing results from other branches of physics, we obtained a new logarithmic law for the susceptibility near the quantum critical point. In conjunction with existing theories, our work provides a fuller understanding of the 'phase diagram' of magnetic droplet systems, and has implications for the design of magnetic thermometers based on giant-moment alloys.

2. Coulomb Blockade Effects in Granular Metals

2003–2006, with Dr. Vikram Tripathi and Dr. Misha Turlakov (Cavendish Laboratory)
Phys. Rev. B 72, 233404 (2005), Phys. Rev. Lett. 96, 046805 (2006), Phys. Rev. B 73, 195113 (2006)

Granular metals often show a soft activation law $\sigma \sim \exp\left[-(T_0/T)^{1/2}\right]$ in their electrical conductivity, in violation of the hard (Arrhenius) activation law $\sigma \sim \exp(-E_\sigma/T)$ predicted by naïve Coulomb blockade theory. There had been a suggestion that the soft-activation law could be understood within the context of the Ambegaokar-Eckern-Schön (AES) model. I studied the AES model analytically and numerically, and found that a hard-activation law persists all the way down to zero temperature in the case of a perfectly regular array of grains. This result is in agreement with recent experiments on silver nanoparticle arrays with controllable disorder, and suggests that the soft-activation law is due to disorder.

Following the above work, we went on to calculate the thermal conductivity of granular metals from the AES model. We showed that whereas charge transport is dominated by charged single-particle excitations, heat transport is dominated by neutral electron-hole pair excitations; thus, whereas the electrical conductivity has an Arrhenius behavior, $\sigma \sim \exp(-E_\sigma/T)$, the thermal conductivity has a power-law behavior, $\kappa \sim T^3$. We thus made the important prediction that the Wiedemann-Franz law ($\kappa \propto T\sigma$) is violated in granular metals, especially in the regime of weak inter-grain tunneling.

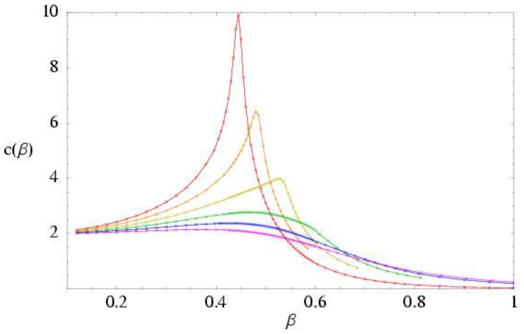

Neutral electron-hole pair cotunneling processes that give rise to a power-law term in the thermal conductivity.

We further studied the optical conductivity, that is, the finite-frequency (AC) electrical conductivity $\sigma(\omega, T)$. We pointed out that optical conductivity contains important contributions from intra-grain polarization oscillations. We generalized the AES action in order to model charging, tunneling and polarization effects in a unified way. This allowed us to calculate the corrections to the optical conductivity due to inter-grain quantum tunneling.

3. Magnetism and Disorder—Random-Bond Ising Models

2006–2007, under Prof. Erica W. Carlson (Purdue University)
Phys. Rev. B 76, 014403 (2007), *Phys. Rev. Lett. 97, 227205 (Nov 2006)*

I developed and implemented a new algorithm for solving planar Ising models with any configuration of bond strengths. It is a generalization of the Frank-Lobb bond-propagation algorithm for resistor networks. It executes in $O(L^3)$ time, on par with the fastest method previously known (Pfaffian method with nested dissection), and for dilute models near percolation it is even faster, taking only $O(L^2 \ln L)$ time. It operates directly in the spin basis, it is massively parallelizable, and it gives fresh insight on the peculiar "hidden integrability" of 2D Ising models and suggests new directions for tackling other problems.



Specific heat of 128x128 realizations of the binary random-bond Ising model for various disorder strengths, computed using our Ising bond propagation algorithm.

4. Superconductivity and Disorder—Inhomogeneous Superconductors

2006–present, under Prof. Erica W. Carlson (Purdue University)
Phys. Rev. B 75, 132506 (2007)

Superconductivity requires two ingredients: the binding of electrons to form Cooper pairs, and the phase coherence of these pairs across the entire sample. The superconducting transition temperature (T_c) is limited by the pairing energy scale T_{pair} as well as the phase-coherence energy scale T_{phase}. In the cuprate superconductors these two scales are comparable, and it can be argued that the rise and fall in T_c as a function of doping is due to a trade-off between T_{pair} and T_{phase}. Experiments now suggest that T_{pair} and T_{phase} are spatially varying and locally anticorrelated. It is important to understand the consequences of this inhomogeneity: does it help or harm superconductivity, or is it a side issue entirely?

We began by studying XY models, which may be a good description of superconductors with low superfluid density (such as the underdoped cuprates) that are dominated by phase fluctuations of the order parameter. We have shown, using "honest", unbiased numerical methods, that *certain* types of inhomogeneity ("frameworks") can significantly increase the T_c of a 2D XY model while preserving the zero-temperature energy density and superfluid stiffness. We support these conclusions with corresponding results for Ising models that are exact and analytic.

We are also studying Ginzburg-Landau models, which allow for amplitude fluctuations and a spatially varying T_{pair}. Our phenomenological approach complements work by others (such as that on Hubbard models with spatially varying U).

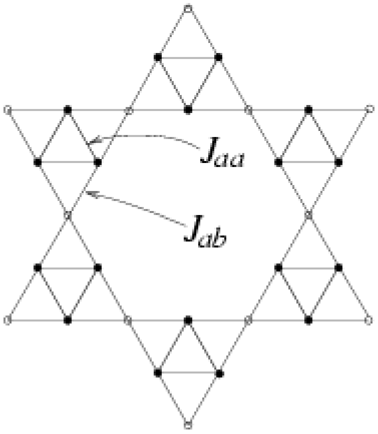
Besides giving an understanding of `naturally occurring' inhomogeneous superconductors, our work raises interesting suggestions for the design of nanostructured composite materials. Given two types of superconductor with complementary characteristics (one with high T_{pair}, the other with high T_{phase}), can one arrange these in a pattern such that the proximity effect can "borrow" the best from each superconductor to produce a macroscopic T_c which exceeds the original T_c's of both components?

5. Geometrically Frustrated Magnetism

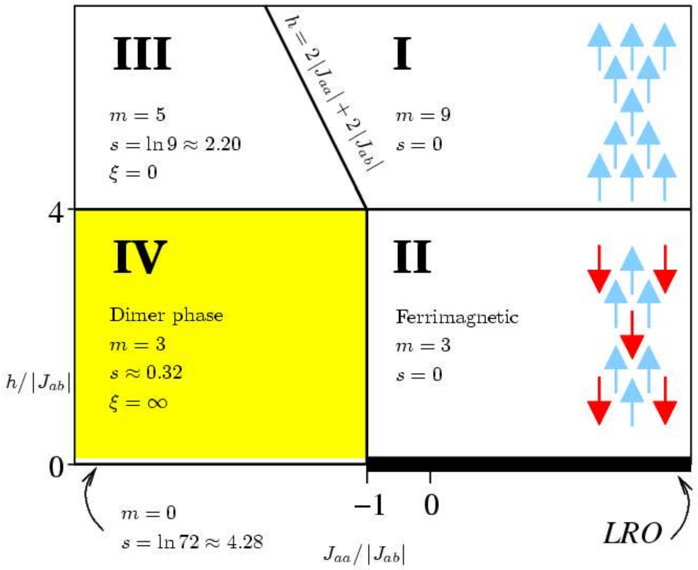
2007–present, with Prof. Erica W. Carlson and Dr. Daoxin Yao (Purdue University)
arXiv:0711.3471

In certain classical spin systems the spins are arranged such that they remain in a disordered, paramagnetic, "spin liquid" state at low temperature despite experiencing strong spin-spin interactions. Such "geometrically frustrated" systems are characterized by residual entropies (macroscopic ground state degeneracies) and interesting short-range or power-law correlations. They have practical applications as magnetic refrigerants, and they have recently received much attention due to the possibility of quantum fluctuations inducing novel phases such as valence bond solids or liquids.

An interesting example of geometric frustration is the class of organic copper salts $Cu_9X_2(cpa)_6 \cdot xH_2O$ (cpa=2-carboxypentonic acid, a derivative of ascorbic acid; X=F,Cl,Br), where the Cu spins form a ``triangular kagome lattice" (TKL). We have performed an extensive study of the Ising model on the TKL, deriving exact analytic expressions for thermodynamic quantities (free energy, internal energy, specific heat, entropy) where possible, and mapping out the entire phase diagram as a function of coupling constants, temperature, and applied magnetic field. We have also obtained Monte Carlo results for the magnetization and susceptibility, which can be compared with experiment to determine whether an Ising description is possible or if a quantum Heisenberg model is required. Our most significant result is that applying a field triggers a transition from a disordered phase to a critical phase with power-law correlations related to the random dimer model on the honeycomb lattice. The finite-field phase is similar to that for the kagome Ising model in a field as described by Moessner and Sondhi (2000-2003), but the zero-field phase differs: whereas the kagome Ising model has exponential correlations and a non-trivial residual entropy, the TKL Ising model has a "perfectly frustrated" state where adjacent triangles completely decouple, giving rise to a residual entropy of ln 72 per unit cell.



The triangular kagome lattice.



Phase diagram of the Ising model on the triangular kagome lattice.

6. Possible Future Research Areas

My work on granular metals is closely related to quantum dots and single-electron transistors, in which Coulomb blockade physics plays an essential role. Together with my work on inhomogeneous superconductivity, it can be extended to granular superconductors—e.g., as described by a random AES model—which experience the combined effects of charging, tunneling, Josephson coupling, and disorder. This topic is relevant not only to the field of high-temperature superconductivity but also to artificial Josephson-junction arrays which have potential applications in quantum computation.

Our Ising bond-propagation algorithm may be applied to 2D dilute Ising models and/or random-bond Ising models, in order to resolve some of the outstanding disagreements about their phase diagrams and critical properties. Random-bond Ising models (RBIMs) are often used to study frustration and spin-glass behavior, and they are closely related to neural networks and information theory.

The logical follow-up to our work on the triangular kagome Ising model would be a study of the XY, Heisenberg, and quantum Heisenberg cases, possibly including longer-ranged interactions. We already have some preliminary results in these directions.

On a more general level, I am fascinated by emergent phenomena, from new types of ground states, elementary excitations and hidden orders in strongly correlated electron materials to complex behavior in classical nonlinear systems. While I believe that understanding the natural world is the primary goal of a physicist, I also look out for possible improvements in theoretical and computational techniques that help in bridging the gap between theory and experiment.

YLL, 2007-11-20