Enhancing Superconductivity Through Inhomogeneity

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Abstract

We show that inhomogeneity can raise T_c in the 2D XY model up to a theoretical maximum of 76% [1]. Our results are relevant to superconductors with low superfluid density, superfluids in nanostructured porous media, and magnetism in inhomogeneous systems.

Introduction

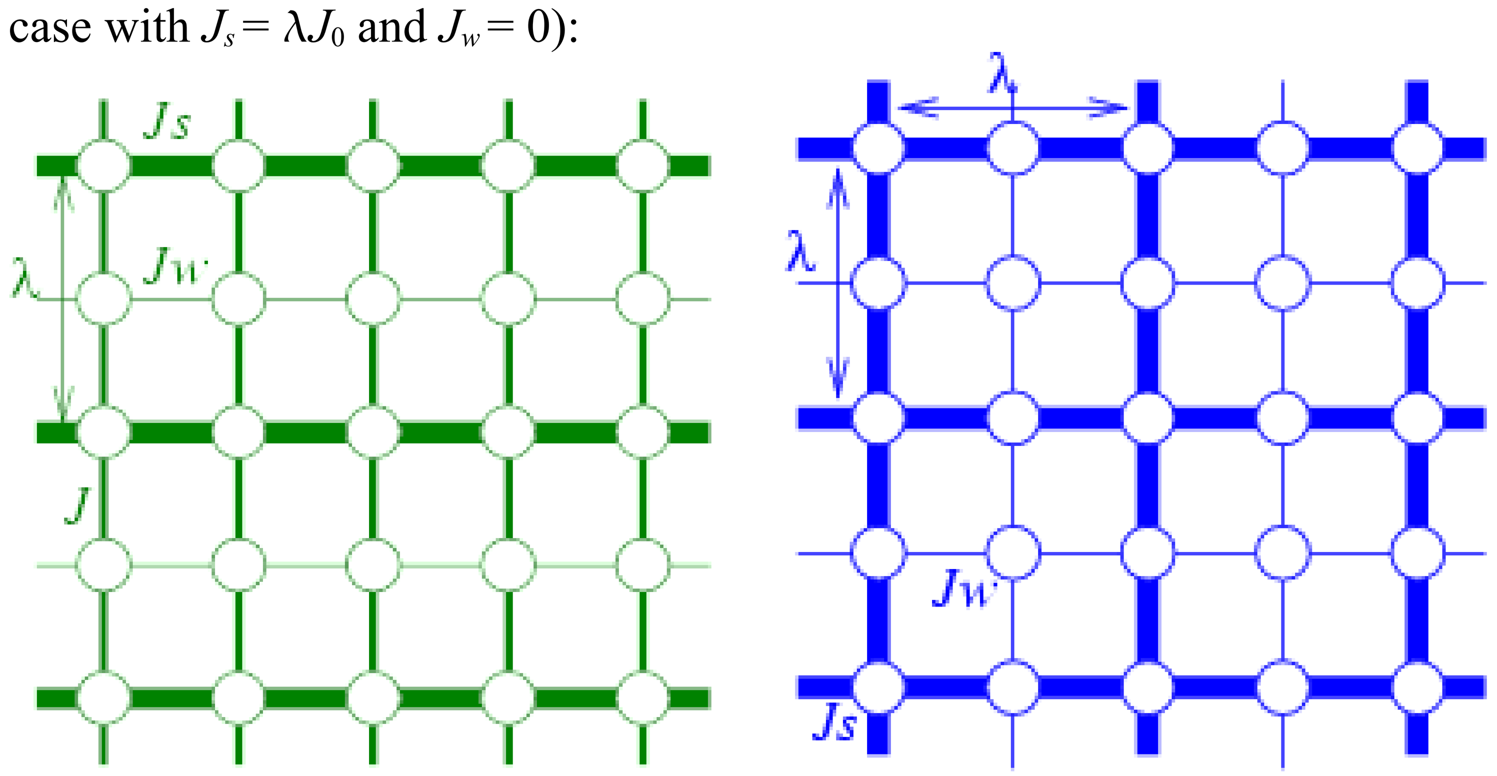
Many strongly correlated systems exhibit **inhomogeneity**, that is, spatial variations in their structure, composition, or local electronic properties. It is important to understand how such mesoscale variation influences macroscopic properties such as superconductivity.

Recent studies [2-4] have shown that the superconducting critical temperature of attractive and repulsive Hubbard models can, in certain situations, be increased by introducing spatial variation in the Hubbard parameters U or t. This inhomogeneity affects both the local Cooper-pairing energy scale and the local phase coherence scale.

To gain physical insight it is useful to focus on one effect at a time. In superconductors with low superfluid density (*e.g.*, underdoped cuprates or Josephson junction arrays), **phase fluctuations** play an important role. [5] The physics of phase fluctuations is captured by XY models. In this work we study 2D XY models with spatially varying couplings, using Monte Carlo methods, focusing on the effect on T_c and on the superfluid density.

Model

The classical 2D XY model consists of phases θ interacting with couplings J_{ij}. We consider spatial variations in J_{ij} that preserve the average coupling J_0. We focus on one- and two-dimensional modulations (in particular, on an extreme case with $J_s = \lambda J_0$ and $J_w = 0$):



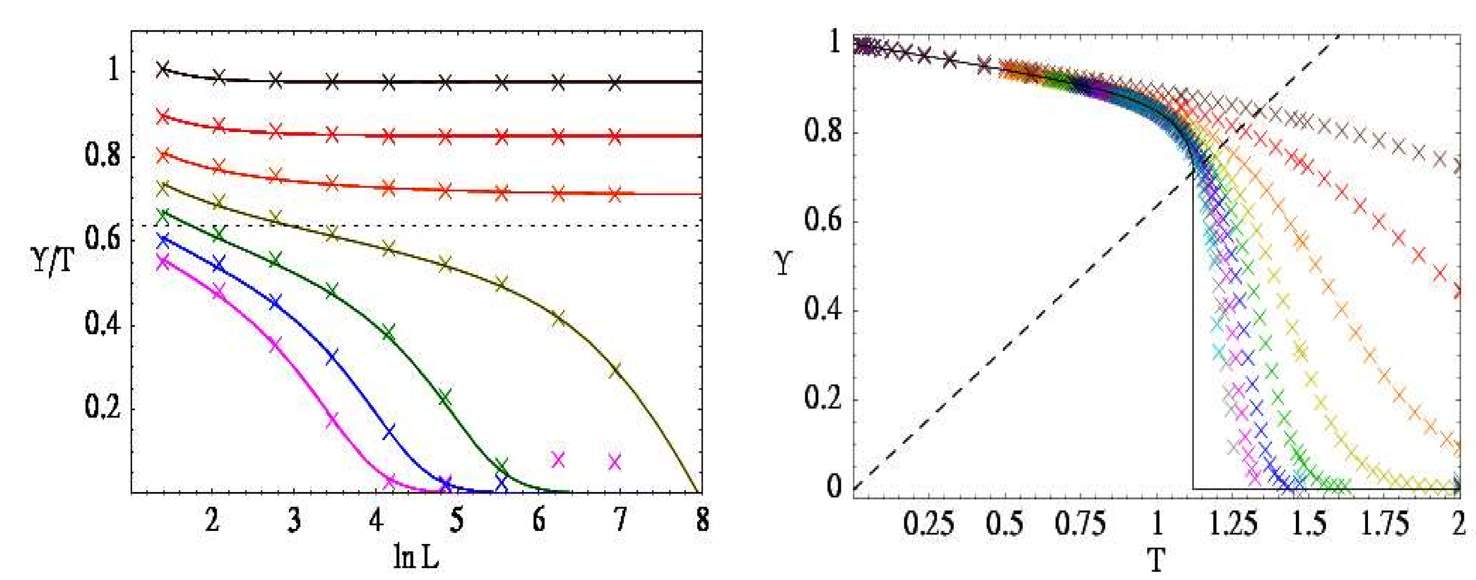
$$H = -\sum_{\langle ij \rangle} J_{ij} \cos(\theta_i - \theta_j)$$

Methods

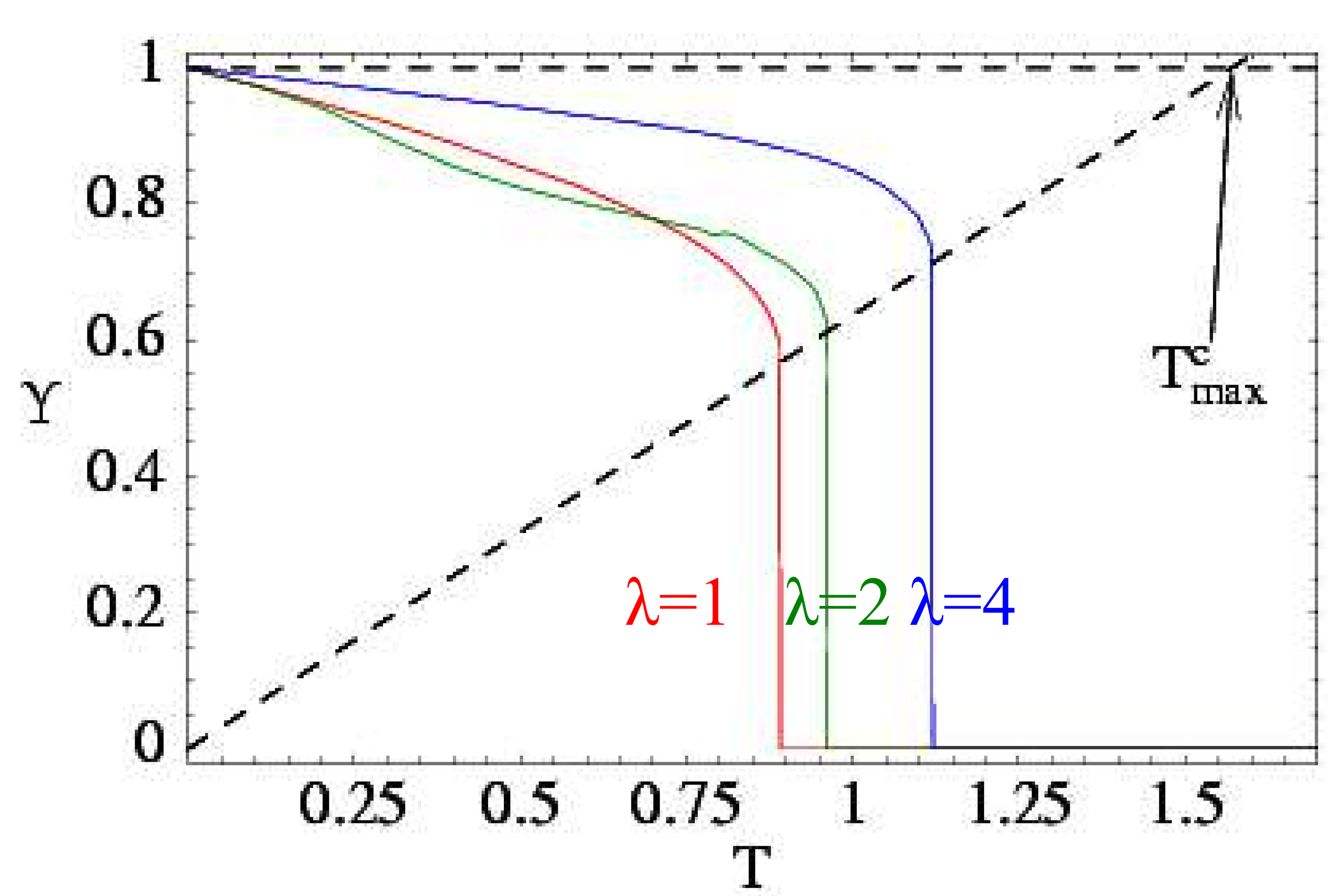
The helicity modulus Υ measures the change in the free energy caused by a small change in the phase angle. It is related to the superfluid density n_s and to the London penetration depth, which can be measured for real superconductors.

We compute $\Upsilon(T, L)$ using Monte Carlo simulations for various temperatures T and system sizes L:

$$\Upsilon = \frac{1}{2} V \left\langle \sum_{\langle ij \rangle} J_{ij} \cos(\theta_i - \theta_j) - \beta \left[\sum_{\langle ij \rangle} J_{ij} \sin(\theta_i - \theta_j) \right]^2 \right\rangle$$



Using finite-size scaling based on the two-parameter flow equations, we extract $\Upsilon(T)$ and the transition temperature T_c.



2D modulations

Why Monte Carlo?

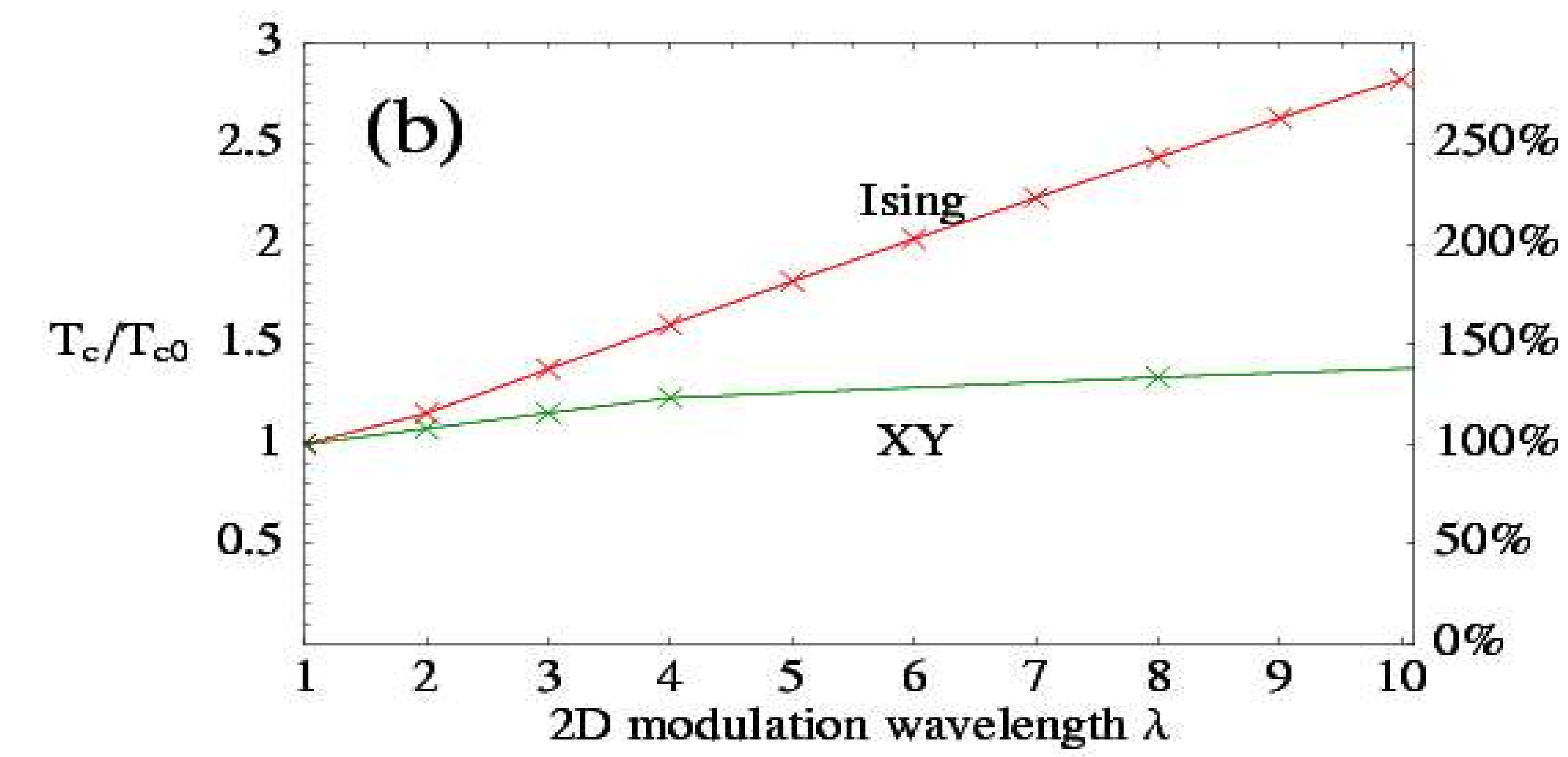
Renormalization methods can be used to compute critical properties of XY models, but they do not give reliable quantitative results for non-universal properties such as T_c. Therefore we have found it necessary to use Monte Carlo with finite-size scaling.

Results

For 2D XY models:

- Random inhomogeneity in the couplings J_{ij} reduces T_c
- One-dimensional modulations reduce T_c
- Two-dimensional modulations **increase** T_c by 10-40%!

This is supported by exact results for inhomogeneous Ising models.



Discussion

Two-dimensional modulations can increase T_c while preserving $\Upsilon(T\!=\!0)$. Thus, the ratio $T_c/\Upsilon(0)$ may be useful for characterizing the degree of inhomogeneity in a superconductor. Using the universal law $\Upsilon(T_c)=0.6365T_c$, we have shown that the maximum increase of T_c is 76%.

The effects of inhomogeneity are also important in other fields. For example, composite materials often have superior mechanical properties compared to pure ones; efficient design of traffic and communications networks often uses links of differing capacities or reliabilities.

Conclusions

Two-dimensional modulations of coupling constants can increase the transition temperature of Ising and XY models. This indicates that certain types of inhomogeneity may result in an enhancement of superconductivity in systems with low superfluid density.

We gratefully acknowledge Purdue University, Purdue Research Foundation, and Research Corporation for support.

References

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