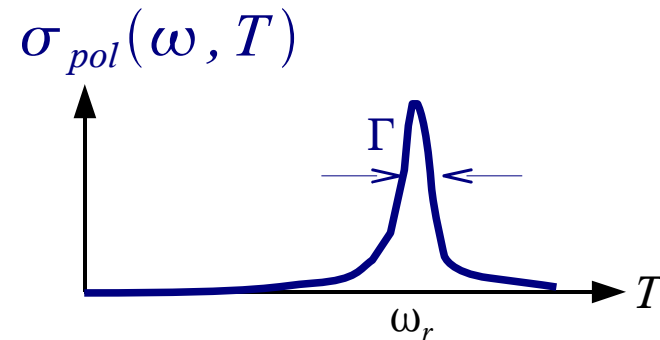
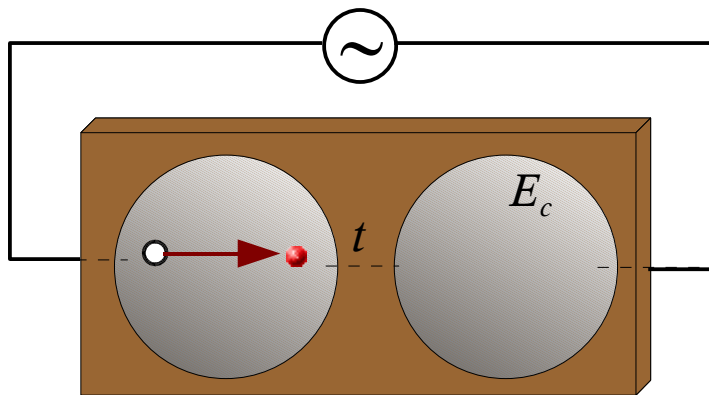


# Optical Conductivity of Granular Metals

Yen Lee Loh<sup>1</sup> and Vikram Tripathi<sup>2</sup>

<sup>1</sup>Physics Department, Purdue University

<sup>2</sup>Theory of Condensed Matter Group, Cavendish Laboratory, Cambridge, UK



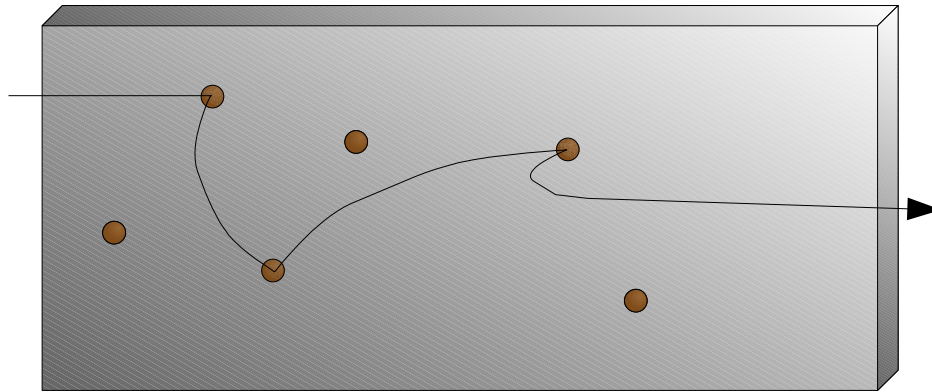
New effective field theory enables calculation of AC properties

AC conductivity is dominated by intra-grain polarization resonance

Width of resonance is related to inelastic cotunneling:

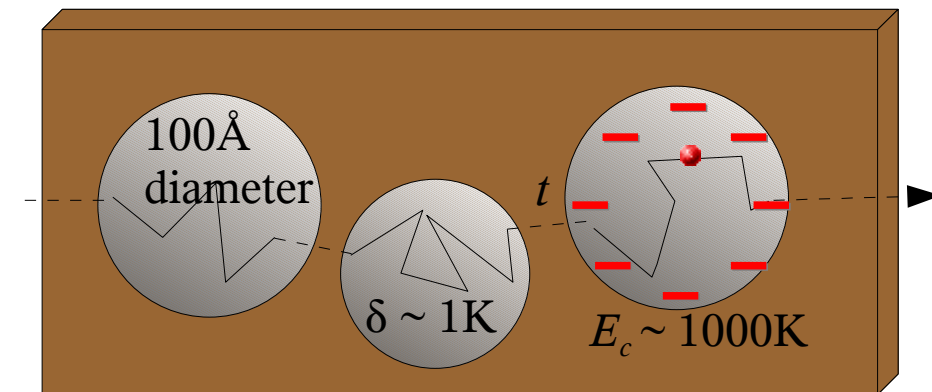
$$\Gamma(\omega, T) \sim g^2 (T^2 + \omega^2) / E_c$$

# Charge Transport in Metal-Insulator Composites



## Dirty metal

Conductivity limited by impurity scattering



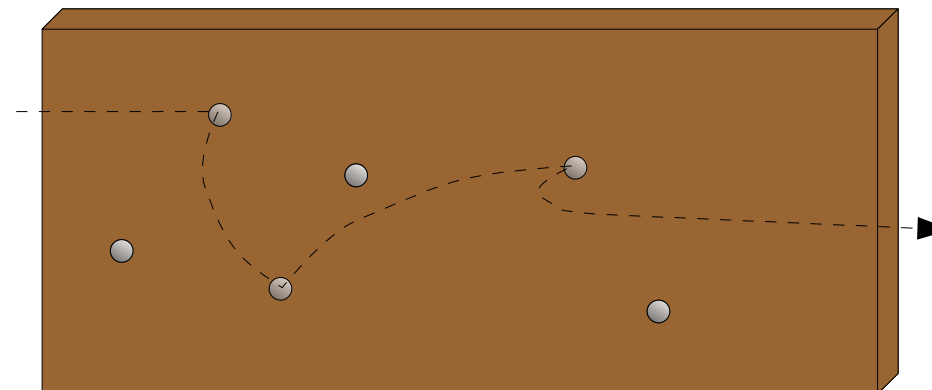
## Granular metal

Tunneling

Near-continuum of energy levels

Coulomb blockade

→ Interesting physics for  $\delta < T < E_c$ !



## Dirty insulator

Conduction due to thermally activated hopping

# Granular metals: DC conductivity

Electron Hamiltonian

$$\hat{H} = \sum_{i\lambda} \xi_{\lambda} c_{i\lambda}^{\dagger} c_{i\lambda} + \sum_{ij\lambda\lambda'} t_{ij} c_{i\lambda}^{\dagger} c_{j\lambda'} + \sum_i E_c Q_i^2$$

*tunneling*                      *charging*

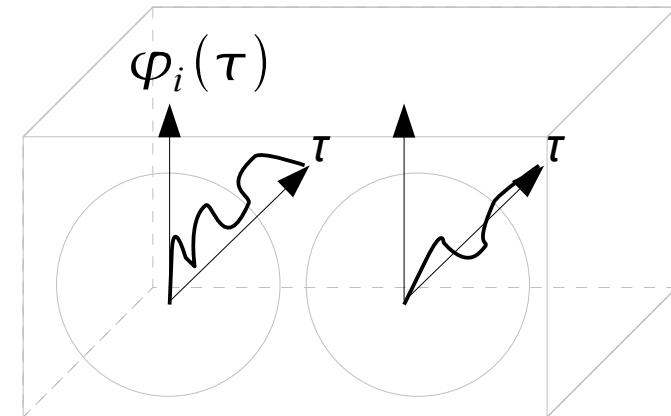
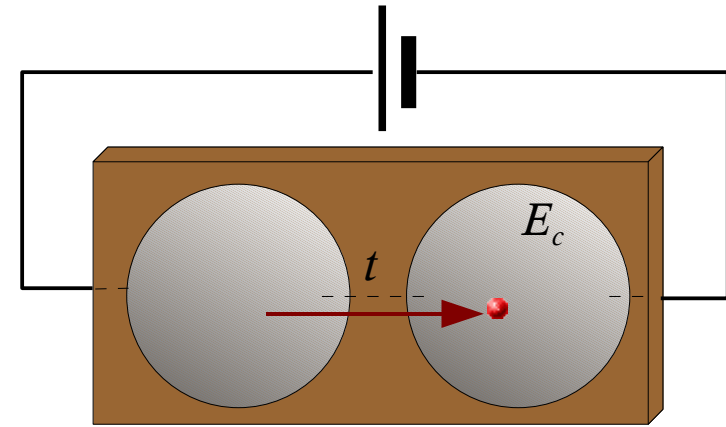
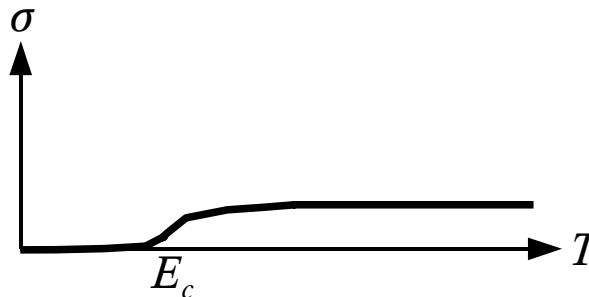
Hubbard-Stratonovich transformation  
Gauge transformation

Ambegaokar-Eckern-Schön phase action

$$S[\varphi] = \int_{\tau\tau'} \sum_{ij} \frac{\pi^2 v^2 t^2}{\sin^2 \pi T \tau} \cos(\varphi_{i\tau} - \varphi_{i\tau'} - \varphi_{j\tau} + \varphi_{j\tau'}) - \int_{\tau=0}^{\beta} \sum_i \frac{1}{4E_c} \left( \frac{d\varphi}{d\tau} \right)^2$$

functional integration  
analytic continuation

$$\sigma(T) = \sigma_0 e^{-E_{eff}/T}$$



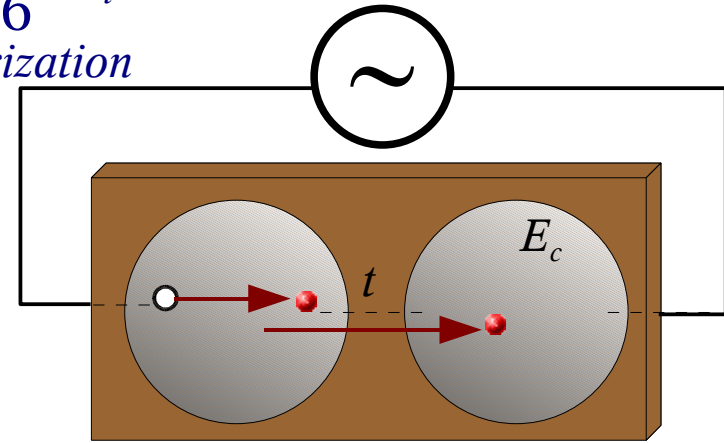
# Granular metals: AC (optical) conductivity

Electron Hamiltonian

$$\hat{H} = \sum_{i\lambda} \xi_\lambda c_{i\lambda}^\dagger c_{i\lambda} + \sum_{ij\lambda\lambda'} t_{ij} c_{i\lambda}^\dagger c_{j\lambda'} + \sum_i E_c Q_i^2 + \sum_i \frac{V}{6} P_i^2$$

tunneling      charging      polarization

Hubbard-Stratonovich transformation  
Gauge transformation



Ambegaokar-Eckern-Schön phase action

$$S_{AES}[\varphi] = \int \sum_{\tau\tau'} \sum_{ij} \frac{\pi^2 v^2 t^2}{\sin^2 \pi T \tau} \cos(\varphi_{i\tau\tau'} - \varphi_{j\tau\tau'}) - \int_{\tau=0}^{\beta} \sum_i \frac{1}{4E_c} \left( \frac{d\varphi}{d\tau} \right)^2$$

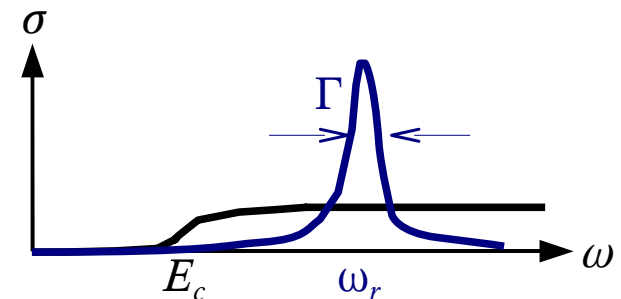
$$S_{pol}[\varphi, \theta] = -b \int \sum_{\tau\tau'} \sum_{ij} \frac{\pi^2 v^2 t^2}{\sin^2 \pi T \tau} \cos(\varphi_{i\tau\tau'} - \varphi_{j\tau\tau'}) [\theta_{i\tau\tau'}^2 + \theta_{j\tau\tau'}^2] - \int_{\tau} \sum_i \frac{1}{4E_c} \left[ \left( \frac{d\theta}{d\tau} \right)^2 + \omega_r^2 \theta^2 \right]$$

$\omega_r^2 = 4\pi n e^2 / 3m$

functional integration  
analytic continuation

$$\sigma(\omega, T) = \sigma_{AES}(\omega, T) + \sigma_{pol}(\omega, T)$$

$$\propto \frac{-i\omega}{\omega^2 - \omega_r^2 - i\Gamma\omega}$$

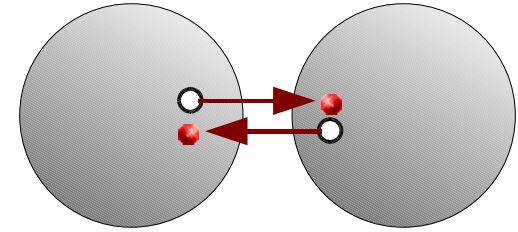


# Summary

Thermal conductivity has direct contribution from **inelastic cotunneling** of neutral pairs  
(Tripathi & Loh, PRL 96, 046805; abstract B35.00007)

$$\kappa(T) \propto \frac{g^2 T^3}{E_c^2}$$

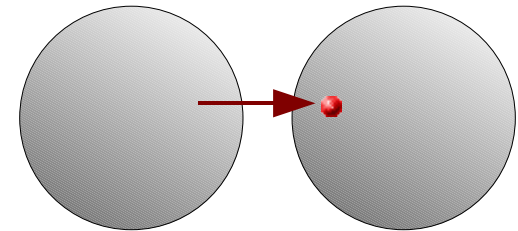
$$g \propto t^2$$



DC conductivity dominated by **inter-grain tunneling**  
Activation energy reduced by inelastic cotunneling

$$\sigma(T) \propto g e^{-E_{eff}/T}$$

$$E_{eff}(g) \approx E_c e^{-\alpha g}$$



AC conductivity dominated by **intra-grain polarization resonance**  
Resonance width related to inelastic cotunneling  
(Tripathi & Loh, cond-mat/0601138)

$$\sigma(\omega, T) \propto -\frac{ine^2 f}{m} \frac{\omega}{\omega^2 - \omega_r^2 - i\Gamma\omega}$$

$$\Gamma(\omega, T) \propto \frac{g^2(T^2 + \omega^2/4\pi^2)}{E_c}$$

