Third harmonic generation by Bloch-oscillating electrons in a quasioptical array

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We compute the third harmonic field generated by Bloch-oscillating electrons in a quasioptical array of superlattices under THz irradiation. The third harmonic power transmitted oscillates with the internal electric field, with nodes associated with Bessel functions in $eEd/\hbar\omega$. The nonlinear response of the array causes the output power to be a multivalued function of the incident laser power. The output can be optimized by adjusting the frequency of the incident pulse to match one of the Fabry-Pérot resonances in the substrate. Within the transmission-line model of the array, the maximum conversion efficiency is 0.1%.
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Miniband transport in semiconductor superlattices is fundamentally nonlinear. In modest fields carriers will be redistributed over the entire miniband suppressing current flow and producing a strongly nonlinear response. If the frequency of the applied electric field exceeds the scattering rate, the quantum nature of the nonlinear transport emerges and has been displayed as (a) Bloch oscillations,[1] (b) dynamic localization,[2] and (c) absolute negative conductance.[3] Strong nonlinearities lead to rich electron dynamics[4,5] complicated by bistable interaction with the radiation field,[6] and naturally invite the exploration of these materials for terahertz harmonic generation.

At terahertz frequencies, the skin effect and parasitics constrain devices to micron dimensions and adequate harmonic power can only be expected by power combining in quasioptical arrays.[7,8] In the following we model the harmonic response of semiconductor superlattices embedded in a quasioptical array. The nonlinear response of the superlattice is obtained in the absence of scattering and therefore relevant to harmonic generation at high terahertz frequencies. In this limit the superlattice may be viewed as a nonlinear ''inductor,'' a current controlled varactor. To approach real array performance the effect of substrate internal reflection and resonances is included. We find that the third harmonic output is a nonmonotonic and sharply multivalued function of the input power. (The features discussed earlier for a wave propagating in an infinite superlattice medium[6] appear for a device in a quasioptical array.) The multivaluedness arises from the nonlinear ''feedback'' effect of the array on the incident field. Not surprising but important is the fact that the output can be optimized by bringing the system in resonance with Fabry-Pérot modes in the substrate.

Third harmonic current. We consider an infinite, one-dimensional superlattice of period d with a single miniband having a tight-binding energy-quasimomentum dispersion $\epsilon_k = -(\Delta/2)\cos(kd)$. Upon irradiation with a time-dependent electric field $E(t)$, the electrons in the superlattice set up a current density that can be obtained[5] by solving the Boltzmann transport equation within a relaxation time approximation for collisions, with a collision time τ

$$j(t) = \frac{\Delta ned}{2\hbar\tau}\int_{-\infty}^{t} dt' e^{-(t-t')/\tau}\sin[\eta(t)-\eta(t')], \qquad (1)$$

where $\eta(t) \equiv \int_{-\infty}^{t} dt' eE(t')d/\hbar$, n is the electron density in the conduction miniband, and e is the electronic charge. For a field $E(t) = E_1\cos\omega t$ in the weak collision limit ($\omega\tau \gg 1$), the electrons execute ac Bloch oscillations, and the current density depends nonlinearly on the field through the parameter $\Theta_1 \equiv eE_1 d/\hbar\omega$. The nonlinear effects are governed by the doping density n, which controls the plasma frequency $\omega_P \equiv \sqrt{4\pi ne^2/m^*\epsilon_0}$, where the effective mass $m^* \equiv 2\hbar^2/\Delta d^2$, and ϵ_0 is the static (dc) dielectric constant of the superlattice. The current density in the superlattice is then given by

$$j(t) = \frac{\epsilon_0\omega_P^2}{2\pi\omega}\sum_{m=0}^{\infty}\left[\frac{J_0(\Theta_1)J_{2m+1}(\Theta_1)}{\Theta_1}\right] \times E_1\sin(2m+1)\omega t. \qquad (2)$$

The above current density is out of phase with the field $E(t)$ and contains only odd harmonics, whose conductances vary as $1/\omega$. Such a response is essentially inductive, except that it is nonlinear in the field parameter Θ_1. In analogy with nonlinear capacitors, we thus model the superlattice in the weak collision limit by a nonlinear inductor. The first and third harmonic current densities from Eq. (2) are shown in Fig. 1. At low fields ($\Theta_1 \ll 1$), expanding $J_3(\Theta_1)$ causes the third harmonic current density to rise cubically with the field.

Quasioptical array. We now incorporate the above current density from the superlattice into an experimentally re-

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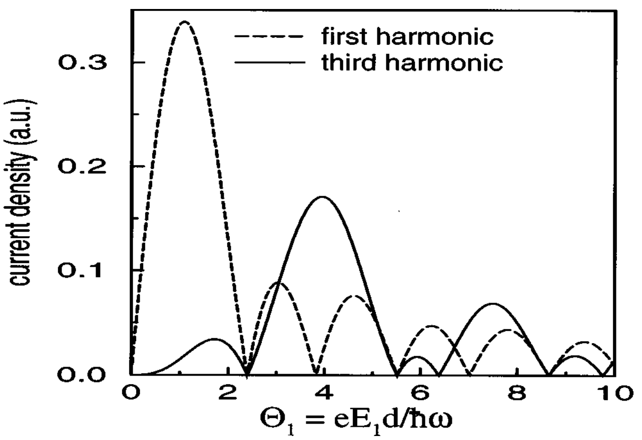

FIG. 1. Absolute value of first and third harmonic currents predicted by Eq. (2), plotted vs $\Theta_1 = eE_1d/\hbar\omega$. The current densities vanish at roots of $J_0(\Theta_1)J_1(\Theta_1)$ and $J_0(\Theta_1)J_3(\Theta_1)$, respectively.

alizable setup, namely the quasioptical array. In our model, an $N\times N(N\sim 10)$ periodic array of mesas, of period 25 μm, sits on a GaAs substrate. Each mesa is 2 μm tall and contains a 5000 Å long GaAs/AlGaAs superlattice of cross section $10\,\mu\mathrm{m}\times 2\,\mu\mathrm{m}$, placed between gold layers (Fig. 2). The mesa itself is sandwiched between gold antennae that redirect the incident electric field along the superlattice growth direction. Since the gold layers are highly conducting, the incident voltage applied across the 25 μm period acts effectively across the 5000 Å superlattice, leading to a field amplification factor of about 50. This enables one to achieve high fields in the superlattice for moderate incident powers.

We model the array by a set of transmission lines as shown in Fig. 2. As long as the array period is smaller than the incident THz wavelength, the array can be modeled as a sheet with properties defined by a single element. The incident and transmitted fields emerge in air, which has an impedance $Z_0 = 377\,\Omega$. The array sits on a GaAs substrate with a dc dielectric constant $\epsilon_0 \approx 13$, and impedance $Z_s = Z_0/\sqrt{\epsilon_0} \approx 100\,\Omega$. The slot capacitance in the figure corresponds to charge building up between the top and bottom antennae, while the mesa capacitance deals with the same effect across the superlattice. The n^+ resistance comes from ohmic contacts placed between the gold layers and the superlattice, while the electronic response in the superlattice acts effectively as a nonlinear inductor, as we have discussed above.

The THz field causes electrons in the nonlinear inductor (superlattice) to execute ac Bloch oscillations. The electronic current from Eq. (1) and the mesa displacement current $(\epsilon_0/4\pi)\partial E/\partial t$ feed through the rest of the circuit. Since the current densities depend directly on the local field Θ_1, we solve the circuit equations for a fixed Θ_1 for the transmitted third harmonic power as well as the corresponding field incident on the array.

Third harmonic power transmitted. In our calculations, the incident and output powers are assumed to be Gaussian distributed over a diameter of 1.5 mm. We choose a frequency $\omega = 2\pi\times 740.3\,\mathrm{GHz}$, and the plasma frequency $\omega_P = 16.4\,\mathrm{THz}$, which for a miniband width $\Delta = 18\,\mathrm{meV}$, and superlattice period $d = 100\,\mathrm{Å}$, corresponds to a heavily doped electron density $n\sim 8\times 10^{16}\,\mathrm{cm}^{-3}$. The slot capacitance is taken to be 7 fF and the n^+ resistance is 2 Ω.

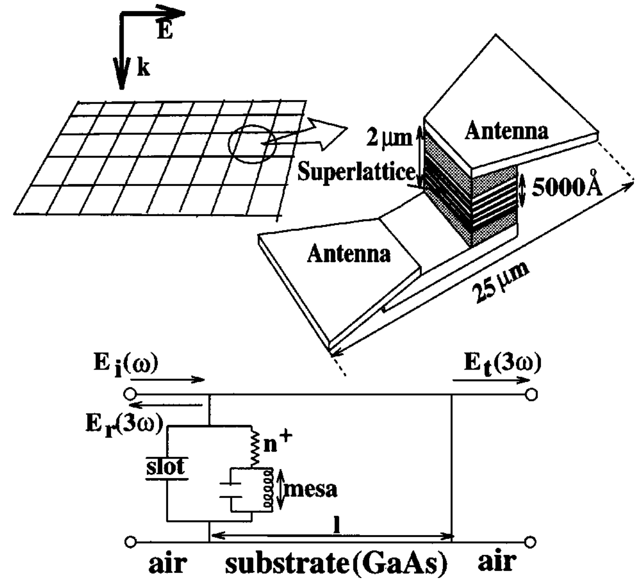

FIG. 2. Schematic of quasioptical array with incident THz laser field. A typical array element is shown magnified. The antenna elements focus the field through the superlattice. Bottom: transmission-line model used to describe the setup. The antenna provide a 7 fF slot capacitance, while the ohmic contacts provide a 2 Ωn^+ resistance. The superlattice itself is modeled by an inductor (except the response is nonlinear in the local field) in series with a mesa capacitance arising from charges building up at the ends of the superlattice.

Figure 3 shows that the third harmonic power transmitted through the substrate depends on the *incident* power nonmonotonically. The nonlinear effects from the device give results qualitatively different from that of an infinite superlattice in Fig. 1. From the above calculations for the higher

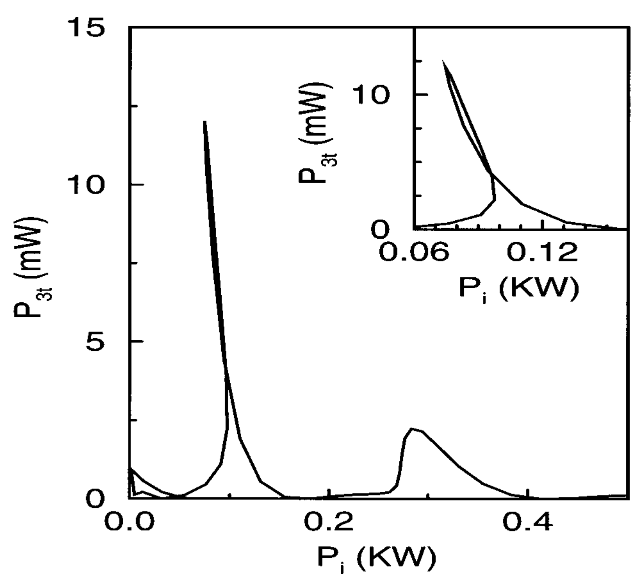

FIG. 3. Third harmonic power transmitted through the array plotted as a function of incident power. Here the applied angular frequency $\omega = 2\pi \times 740.3\,\mathrm{GHz}$, and the substrate thickness is 0.5 mm. The plasma angular frequency $\omega_P = 16.4\,\mathrm{THz}$ corresponds to a doping density of $8\times 10^{16}\,\mathrm{cm}^{-3}$. The third harmonic power varies as $J_0(eEd/\hbar\omega)J_3(eEd/\hbar\omega)$, where E is the local first harmonic field inside the superlattice. The transformation between field inside and outside the superlattice is highly nonlinear, so that a given incident power yields multiple solutions to the field E inside the superlattice. This leads to the bistable structure resolved in the inset.

harmonic currents for a superlattice, the generated third harmonic power is expected to depend on the *local* field Θ_1 as $J_0^2(\Theta_1)J_3^2(\Theta_1)$, the overall factor of $J_0^2(\Theta_1)$ arising from dynamic localization.[2] The transformation from the local field inside the mesa to the incident field is strongly nonlinear, and incorporates the feedback effects of the quasioptical array on the incident field. The minima are thus at incident field values nonlinearly connected to the Bessel functions. Moreover, the nonlinear transformation between the incident and local fields leads to a multistable behavior;[6] the transmitted third harmonic power is a multivalued function of the input laser power, as has been resolved in the inset in Fig. 3. The cubic regime alluded to previously occurs at incident powers too small to be clearly visible in the figure.

Resonances with substrate thickness. The quasioptical array is usually fabricated on a substrate. The third harmonic power generated in the superlattice sets up standing waves inside the substrate. Such Fabry-Pérot effects can be utilized to control the third harmonic power transmitted through the substrate. As we vary the frequency of the incident laser source (or the width of the substrate), the first and third harmonics are brought in and out of Fabry-Pérot resonances with the substrate. This can vary the transmitted third harmonic power by almost two orders of magnitude. For optimal output, we need to maximize the amount of first harmonic at the superlattice responsible for third harmonic generation, as well as the amount of generated third harmonic that transmits through the substrate. Note that even though the first and third harmonic currents are comparable at the superlattice (Fig. 1), the third harmonic generation is only moderately efficient. The maximum power conversion efficiency is observed to be about 0.1% near a Fabry-Pérot resonance ($\omega = 2\pi \times 723.8\,\mathrm{GHz}$) for a 0.5 mm GaAs substrate. Bringing the incident frequency near plasma resonance did not produce any noticeable increase in third harmonic output. The harmonic currents in the quasioptical array are large and comparable to the fundamental currents. This suggests a careful exploration of impedance matching at the fundamental and harmonics may lead to improved efficiencies.

Harmonic generation from superlattices excited by internal reflection in a prism geometry[9] has been reported. More recent work has been carried out in quasioptical arrays.[10] In neither case does bistability appear. However, in both cases the fundamental frequency and material parameters are such that $\omega\tau \approx 1$ and the condition $\omega\tau \gg 1$ are not satisfied. Higher frequencies or improved scattering times are probably necessary to explore bistability in these systems.

In summary, we have addressed terahertz harmonic generation by Bloch-oscillating electrons in a semiconductor superlattice. The successful application of these elements to terahertz generation will require power combining and we have treated the interaction with the electromagnetic field by modeling a plane wave incident on a quasioptical array on a semiconductor substrate. The harmonic response exhibits remarkable bistable features but the efficiency is low. Increased efficiency and harmonic power may follow optimized coupling at the fundamental and harmonic frequency.

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