

Electron-electron relaxation in heterostructures

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Stimulated by recent data for the electron-electron relaxation time measured in two different double-quantum-well systems we consider both intralayer and interlayer screened electron-electron interactions and include a contribution beyond the simple golden-rule result. In particular, we find that (i) including a nongolden-rule contribution of the same order in the interaction always reduces the rate, (ii) as a result, our intralayer golden-rule contribution is significantly smaller than all published calculations, and (iii) while including interlayer scattering may increase the rate, it is still less than the measured rate in a wide double-quantum-well system. [S0163-1829(97)50712-0]

The electron-electron inelastic-scattering time is a very important characteristic of an electron system in metals and semiconductors. Considerable attention has been directed to studying the electron-phase relaxation time in impure low-dimensional systems both theoretically and experimentally. The magnetoconductivity due to the weak localization and the interaction effects, the mesoscopic conductance fluctuations, and the Aharonov-Bohm effects are some of the phenomena which depend on the electron-phase relaxation time.¹

Recent progress in heterostructure growth technology also initiated interest in the electron-electron interaction in pure two-dimensional electron systems.²⁻⁴ Theoretically, the effect of the electron-electron interaction on the electron lifetime was studied in Refs. 5-11. While there was a reasonable agreement between theoretical predictions and the experimental data for zero temperatures² in single-quantum-well systems, the comparison of the data of the experiments in double-quantum-well-systems for finite temperatures^{3,4} is more complicated. In strongly-coupled quantum wells (with wells separation $b=14 \text{ \AA}$) the theory⁴ which takes into account the formation of the electron bound states due to quantum tunneling describes the experimental data quite well. In weakly-coupled quantum wells ($b=175-340 \text{ \AA}$), for which it is possible³ to ignore the formation of the electron bound states, theoretical calculations disagree among themselves, and all of them give the inverse electron lifetimes less than the experimental data. The authors of Ref. 4 also reported disagreement between their theory and the experiment for $b=40 \text{ \AA}$.

The purpose of our paper is to reconsider the problem of the electron relaxation in a single-layer electron system and then study in detail the electron relaxation in a weakly-coupled two-layer electron system taking into account modification of the electron screening and different channels of the electron relaxation.

We use the Keldysh diagram technique for inequilibrium processes in which, besides retarded and advanced electron Green's functions,

$$G^R(P)=[G^A(P)]^*=\frac{1}{\epsilon-\xi_p+i0}, \quad (1)$$

$$\xi_p=\frac{p^2-p_F^2}{2m}, \quad P=(\mathbf{p}, \epsilon),$$

the kinetic Green's function is introduced,

$$G^C(P)=[2n(P)-1][G^A(P)-G^R(P)], \quad (2)$$

where p_F is the Fermi momentum, m is the electron mass, and $n(P)$ is the electron Fermi distribution function. For the problem of the electron-energy relaxation, $n(P)=n(\epsilon)$. The kinetic electron self-energy Σ^C satisfies an equation similar to Eq. (2). The electron collision integral $I(P)$, the kinetic equation that defines the electron-energy relaxation, and the relaxation time τ_{e-e} are determined from the equations (see, e.g., Ref. 12)

$$I(P)=-i\{\Sigma^C(P)-[2n(\epsilon)-1][\Sigma^A(P)-\Sigma^R(P)]\}, \quad (3)$$

$$\frac{dn(\epsilon)}{dt}=\frac{1}{\pi\nu}\int\frac{d\mathbf{p}}{(2\pi)^3}\text{Im}[G^A(P)]I(P), \quad (4)$$

$$\frac{1}{\tau_{e-e}(\epsilon,T)}=-\frac{\partial}{\partial n(\epsilon)}\frac{dn(\epsilon)}{dt}, \quad (5)$$

where ν is the electron density of states. The electron self-energy in the random-phase approximation (RPA) is the first diagram shown in Fig. 1. The corresponding equation for the energy relaxation time in the two-dimensional case is

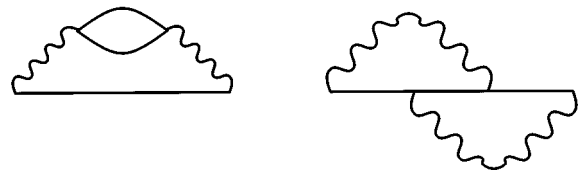


FIG. 1. The random-phase approximation (golden-rule), and the nonrandom-phase approximation (nongolden-rule) diagram of the electron self-energy in the second order in the electron-electron interaction contributing to the electron relaxation.

$$\begin{aligned} \frac{1}{\tau_{e-e}^{(1)}(T, \epsilon)} = & -\frac{8}{\pi\nu} \int \frac{d\mathbf{p}}{(2\pi)^2} \int \frac{dQ}{(2\pi)^3} [N(\omega) + n(\omega + \epsilon)] \\ & \times \text{Im}G^A(P)\text{Im}G^A(P+Q) \\ & \times \text{Im}\Pi^R(Q)|V(Q)|^2, \quad Q=(\mathbf{q}, \omega). \end{aligned} \quad (6)$$

The equilibrium Bose and Fermi distribution functions $N(\omega)$ and $n(\epsilon)$ both depend on the electron temperature T . The retarded scalar-polarization operator is

$$\Pi^R(Q) = -\nu \left(1 + i \frac{\omega}{qv_F} \right), \quad \omega < qv_F, \quad q < 2p_F, \quad (7)$$

where $\nu = m/\pi$ is the two-spin electron density of states. The Coulomb potential screened in the RPA is

$$V(q) = \frac{1}{\nu} \frac{\kappa}{q + \kappa}, \quad \kappa = 2\pi e^2 \nu / \epsilon, \quad (8)$$

where κ is the two-dimensional Debye screening momentum and ϵ is the static dielectric constant.

Intralayer scattering rate

Equation (6) may be rewritten in the following form:

$$\begin{aligned} \frac{1}{\tau_{e-e}^{(1)}} = & \frac{16}{\pi^2 \nu} \int \frac{d\omega}{2\pi} \omega [N(\omega) + n(\epsilon + \omega)] \int \frac{d^2q}{(2\pi)^2} [V(q)]^2 \\ & \times \left[\int \frac{d^2p}{(2\pi)^2} \text{Im}G^A(P)\text{Im}G^A(P+Q) \right]^2. \end{aligned} \quad (9)$$

The integral of the two Green's functions yields $\nu\pi/(2v_Fq)$ and sets the limits for the q integral: $\omega/v_F \leq q \leq 2p_F$. Continuing the calculation of Eq. (9) yields

$$\begin{aligned} \frac{1}{\tau_{e-e}^{(1)}(T, \epsilon)} = & \frac{2}{\pi^2 \nu^2} \int_0^\infty d\omega \omega [N(\omega) + n(\omega + \epsilon)] \\ & \times \int_{q_0}^{2p_F} \frac{dq}{q} [V(q)]^2 \end{aligned} \quad (10)$$

where $q_0 = \omega/v_F$, and finally,

$$\begin{aligned} \frac{1}{\tau_{e-e}^{(1)}(T)} \equiv & \frac{1}{\tau_{e-e}^{(1)}(T, 0)} = \frac{\pi T^2}{4\epsilon_F} \left[\ln\left(\frac{4\epsilon_F}{T}\right) - \ln\left(\frac{2p_F + \kappa}{\kappa + k_0}\right) \right. \\ & \left. - \frac{(2p_F - k_0)\kappa}{(2p_F + \kappa)(\kappa + k_0)} \right], \end{aligned} \quad (11)$$

where $k_0 = T/v_F$.

Equation (10) derived from the RPA self-energy diagram may be also obtained by the second-order perturbation theory in the electron-electron interaction using the golden rule. Thus we later refer to Eq. (11) as the golden-rule contribution. The coefficient $\pi/4$ in the right-hand-side (rhs) of Eq. (13) coincides with Refs. 10 and 11 and differs from all other published works. References 7 and 9 have $1/2\pi$, and Ref. 8 has $\pi/2$. We also note that besides the coefficient, our golden-rule contribution [Eq. (11)] differs from the corresponding equation of Ref. 7 by having only negative nonlogarithmic temperature-dependent terms [the second and the

third terms in the rhs of Eq. (11)], while Ref. 7 has some positive terms which lead to overestimation of the electron relaxation.³

In the three-dimensional case, RPA is valid for high-electron-density $\kappa \ll p_F$ (see, e.g., Ref. 9). In the two-dimensional case, the fact that the $T^2 \ln T$ term in Eq. (11) does not depend on the screening momentum κ means that, unlike the three-dimensional case, terms beyond the golden rule in the electron relaxation may be equally important for any value of the ratio κ/p_F . The first nongolden-rule self-energy, also of second order in V , is the second diagram shown in Fig. 1. The corresponding electron-electron relaxation time may be presented in the form

$$\begin{aligned} \frac{1}{\tau_{e-e}^{(2)}(T, \epsilon)} = & -\frac{8}{\pi^2 \nu} \int \frac{d\omega}{2\pi} \omega [N(\omega) + n(\epsilon + \omega)] \\ & \times \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2q}{(2\pi)^2} \int \frac{d^2q_1}{(2\pi)^2} V(q)V(q_1)\text{Im}G^A(P) \\ & \times \text{Im}G^A(P+Q)\text{Im}G^A(P+Q_1)\text{Im}G^A(P+Q+Q_1). \end{aligned} \quad (12)$$

Calculations show that logarithmic terms in $1/\tau_{e-e}^{(2)}$ differ from that of $1/\tau_{e-e}^{(1)}$ by a factor $-\frac{1}{2}$, due to absence of a factor -2 associated with the electron loop in $1/\tau_{e-e}^{(1)}$, and there are no nonlogarithmic temperature-dependent terms in $1/\tau_{e-e}^{(2)}$. The combined result of both contributions $\tau_{e-e}^{(1)}$ and $\tau_{e-e}^{(2)}$ is

$$\begin{aligned} \frac{1}{\tau_{e-e}(T)} = & \frac{\pi T^2}{8\epsilon_F} \left[\ln\left(\frac{4\epsilon_F}{T}\right) - \ln\left(\frac{2p_F + \kappa}{\kappa + k_0}\right) \right. \\ & \left. - \frac{2(2p_F - k_0)\kappa}{(2p_F + \kappa)(\kappa + k_0)} \right], \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{1}{\tau_{e-e}(\epsilon)} \equiv & \frac{1}{\tau_{e-e}(0, \epsilon)} = \frac{\epsilon^2}{8\pi\epsilon_F} \left[\ln\left(\frac{4\epsilon_F}{\epsilon}\right) - \ln\left(\frac{2p_F + \kappa}{\kappa + k_1}\right) \right. \\ & \left. - \frac{2(2p_F - k_1)\kappa}{(2p_F + \kappa)(\kappa + k_1)} \right], \end{aligned} \quad (14)$$

where $k_1 = \epsilon/v_F$.

All previous works neglected the nongolden-rule diagram and thus overestimated the result by a factor of 2 [compare Eqs. (11) and (13)]. This is the reason why the coefficient in the right-hand-side of Eq. (14) is half the corresponding coefficient in Refs. 7 and 9.

The temperature- and energy-dependent intralayer scattering rates, Eqs. (13) and (14), respectively, are valid in both the low-density $\kappa \gg p_F$ and in the high-density $\kappa \ll p_F$ limits, as has been checked by numerical evaluation of Eqs. (9) and (12). Thus, the observation that the $T^2 \ln T$ term in the electron-relaxation-rate is independent of the screening parameter κ , forces a consideration of nongolden-rule contributions.

Intralayer and interlayer scattering rate

The electron lifetime was measured in Ref. 3 in a system of two parallel two-dimensional electron planes. The dis-

tance between the electron planes was $b=175\text{--}350\text{ \AA}$. In this situation it is important to take into consideration the interaction between electrons in different planes, which modifies screening of interlayer and intralayer electron-electron interaction and also provides an additional channel of electron relaxation.

The nonscreened Coulomb interaction between electrons in different planes separated by the media with the dielectric constant ϵ_1 is

$$U^0(q) = \frac{1}{\epsilon_1} \frac{2\pi e^2}{q} \exp(-qb). \quad (15)$$

Further we will assume for simplicity that $\epsilon \approx \epsilon_1$.

For two identical layers the screened-interlayer potential U and screened-intralayer potential V are described by the equations

$$U = U^0 + V^0 \Pi U + U^0 \Pi V, \quad V = V^0 + V^0 \Pi V U^0 \Pi U. \quad (16)$$

The solution of Eq. (16) is

$$U(q) = \frac{\kappa}{\nu} \frac{k \exp(-qb)}{q + 2\kappa + \kappa^2/q[1 - \exp(-2qb)]},$$

$$V(q) = \frac{\kappa}{\nu} \frac{1 + \kappa/q[1 - \exp(-2qb)]}{q + 2\kappa + k^2/q[1 - \exp(-2qb)]}. \quad (17)$$

Because small momentum transfers are important for the electron-electron relaxation we will assume further that $qb \ll 1$ to get

$$U(q) = \frac{\kappa}{\nu} \frac{1}{q + 2\kappa(1 + \kappa b)}, \quad V(q) = \frac{1}{\nu} \frac{\kappa(1 + 2\kappa b)}{q + 2\kappa(1 + \kappa b)}. \quad (18)$$

Calculating the electron-electron relaxation in a double-well system we note that the screened-intralayer electron-electron potential V contributes to both golden rule and

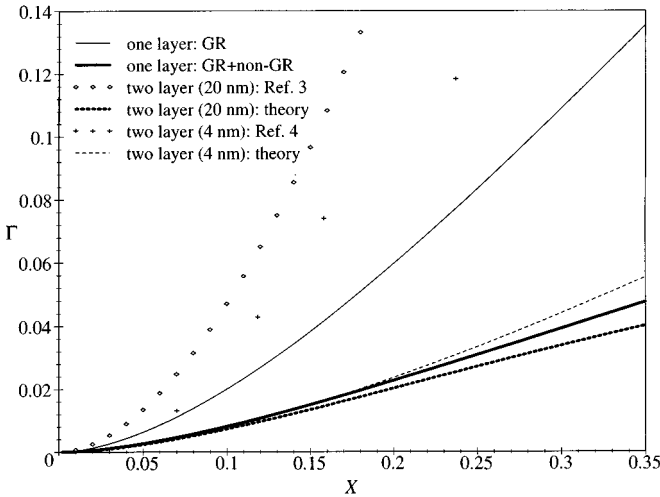


FIG. 2. Dimensionless electron relaxation rate $\Gamma = [\tau_{e-e}(x)\epsilon_F]^{-1}$, $x = T/\epsilon_F$. Experimental data are from Refs. 3 and 4. Theoretical one-layer golden-rule data are from Eq. (11), one-layer golden-rule and non-golden-rule data are from Eq. (13), and two-layer data are from Eq. (19).

TABLE I. Comparison of experimental (Refs. 3 and 4) and theoretical values of the dimensionless electron relaxation rate $\Gamma = [\tau_{e-e}(x)\epsilon_F]^{-1}$, $x = T/\epsilon_F$ for $x=0.1$.

Experiment	ϵ_F (meV)	b (\AA)	$p_F b$	κb	$p b$	$\Gamma(0.1)$ expt.	$\Gamma(0.1)$ theor.
Ref. 3	5.7	200	2	4	1	0.048	0.0073
Ref. 4	10.9	40	0.55	0.8	1	0.031	0.0077

nongolden-rule diagrams while the interlayer potential U contributes only to the golden-rule diagram. The result of an analytical calculation using Eq. (18) is

$$\frac{1}{\tau_{e-e}(T)} = \frac{\pi T^2}{32\epsilon_F(1 + \kappa b)^2} \left\{ [(1 + 2\kappa b)^2 + 2] \right.$$

$$\times \left[\ln\left(\frac{2\epsilon_F}{T} \frac{p b}{p_F b}\right) - \ln\left(\frac{p b + 2\kappa b(1 + \kappa b)}{k_0 b + 2\kappa b(1 + \kappa b)}\right) \right]$$

$$\left. - \frac{4[(1 + 2\kappa b)^2 + 1]\kappa b(p b - k_0 b)(1 + \kappa b)}{[p b + 2\kappa b(1 + \kappa b)][k_0 b + 2\kappa b(1 + \kappa b)]} \right\},$$

$$p b = \min\{2p_F b, 1\}. \quad (19)$$

Equation (19) is valid for arbitrary electron density and arbitrary relation between the parameters $p_F b$ and κb . It is important to keep the low-momentum cutoff k_0 not only in $T^2 \ln T$ terms, but also in the T^2 terms to keep τ_{e-e} positively defined, e.g., for $T=20\text{ K}$ and $\epsilon_F=60\text{ K}$, we cannot ignore $k_0=0.3p_F$. The analytic solution, Eq. (19), is possible only for simplified forms of the potentials, Eq. (18), instead of the exact forms, Eq. (17). Numerical estimates show that this approximation is reasonable.

Using the data reported in Refs. 3 and 4, we presented in Fig. 2 the results of theoretical calculations in a broad temperature range and in Table I experimental and theoretical data for one particular temperature $T=0.1\epsilon_F$. From Table I and from Fig. 2 we see that our theoretical calculations [Eq. (19)] disagree with experimental data. The reason for this disagreement is not clear. Probably the tunnel-resonance width of Ref. 3 and the resistance-resonance width of Ref. 4 have a contribution of nonrelaxation origin. Note that ‘‘excellent agreement’’ reported in recent papers^{10,11} seems too optimistic. In particular, specifics of the electron relaxation in a two-layer system, the interlayer relaxation, and the non-golden-rule contribution, which, as have been shown above, diminishes the result by at least a factor of 2, were ignored.

At very low temperatures, when $\ln(4\epsilon_F/T) \gg 1$, higher-order terms should be considered. This problem was studied by the T -matrix approximation for the Hubbard model in Ref. 13 and recently for the Coulomb interaction in Ref. 14. It was shown in Ref. 14 that at very low temperatures when only logarithmic terms are important higher-order terms lead to effectively diminishing the coefficient to the $T^2 \ln T$ term by another factor of 2 in comparison with Eq. (13).

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- ¹B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. Polak (North-Holland, Amsterdam, 1985).
- ²A. Yacoby, U. Sivan, C. P. Umbach, and J. M. Hong, *Phys. Rev. Lett.* **66**, 1938 (1991); A. Yacoby, M. Heiblum, H. Strikman, V. Umansky, and D. Mahalu, *Semicond. Sci. Technol.* **9**, 907 (1994).
- ³S. Q. Murphy, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, *Phys. Rev. B* **52**, 14 825 (1996).
- ⁴M. Slutzky, O. Entin-Wohlman, Y. Berk, A. Palevski, and H. Shtrikman, *Phys. Rev. B* **53**, 4055 (1996).
- ⁵C. Hodges, H. Smith, and J. W. Wilkins, *Phys. Rev. B* **4**, 302 (1971).
- ⁶A. V. Chaplik, *Zh. Eksp. Teor. Fiz.* **60**, 1845 (1971) [*Sov. Phys. JETP* **33**, 997 (1971)].
- ⁷G. F. Giuliani and J. Quinn, *Phys. Rev. B* **26**, 4421 (1982).
- ⁸H. Fukuyama and E. Abrahams, *Phys. Rev. B* **27**, 5976 (1983).
- ⁹J. Rammer and H. Smith, *Rev. Mod. Phys.* **58**, 323 (1986).
- ¹⁰L. Zheng and S. Das Sarma, *Phys. Rev. B* **53**, 9964 (1996).
- ¹¹T. Jungwirth and A. H. MacDonald, *Phys. Rev. B* **53**, 7403 (1996).
- ¹²M. Yu. Reizer, *Phys. Rev. B* **39**, 1602 (1989).
- ¹³H. Fukuyama and Y. Hasegawa, *Prog. Theor. Phys. Suppl.* **101**, 441 (1990).
- ¹⁴D. Menashe and B. Laikhtman, *Phys. Rev. B* **54**, 11 561 (1996).