Electron-electron relaxation in heterostructures

Michael Reizer and John W. Wilkins
Department of Physics, The Ohio State University, Columbus, Ohio 43210-1106
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Stimulated by recent data for the electron-electron relaxation time measured in two different double-quantum-well systems we consider both intralayer and interlayer screened electron-electron interactions and include a contribution beyond the simple golden-rule result. In particular, we find that (i) including a nongolden-rule contribution of the same order in the interaction always reduces the rate, (ii) as a result, our intralayer golden-rule contribution is significantly smaller than all published calculations, and (iii) while including interlayer scattering may increase the rate, it is still less than the measured rate in a wide double-quantum-well system. [S0163-1829(97)50712-0]

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The coefficient $p$ where $k$ and $q$ both depend on the electron temperature $T$. The retarded scalar-polarization operator is

$$\Pi(R) = -\nu \left( 1 + i \frac{\omega}{qq_F}, \quad \omega < qq_F, \quad q < 2p_F, \right)$$

where $\nu = m/\pi$ is the two-spin electron density of states. The Coulomb potential screened in the RPA is

$$V(q) = \frac{\kappa}{\nu q + \kappa}, \quad \kappa = 2\pi e^2 \nu / \epsilon,$$

where $\kappa$ is the two-dimensional Debye screening momentum and $\epsilon$ is the static dielectric constant.

**Intralayer scattering rate**

Equation (6) may be rewritten in the following form:

$$\frac{1}{\tau_{c,e}^{(1)}(T,\epsilon)} = \frac{16}{\pi \nu} \int \frac{\omega}{2\pi} \omega \left[ N(\omega) + n(\epsilon + \omega) \right] \int \frac{d^2 q}{(2\pi)^2} [V(q)]^2$$

$$\times \left[ \int \frac{d^2 p}{(2\pi)^2} \text{Im} G^A(P) \text{Im} G^A(P + Q) \right]^2.$$

The integral of the two Green’s functions yields $\nu \pi/(2v_Fq)$ and sets the limits for the $q$ integral: $\omega/v_F \leq q < 2p_F$. Continuing the calculation of Eq. (9) yields

$$\frac{1}{\tau_{c,e}^{(1)}(T,\epsilon)} = \frac{2}{\nu v_F} \int_0^\infty d\omega \omega \left[ N(\omega) + n(\epsilon + \omega) \right]$$

$$\times \left[ \int \frac{d^2 q}{q} [V(q)]^2 \right].$$

where $q_0 = \omega/v_F$, and finally,

$$\frac{1}{\tau_{c,e}^{(1)}(T)} = \frac{1}{\tau_{c,e}(T,0)} = \frac{\pi T^2}{4\epsilon_F} \left[ \ln \left( \frac{4\epsilon_F}{T} \right) - \ln \left( \frac{2p_F + \kappa}{\kappa + k_0} \right) \right]$$

$$- \frac{(2p_F - k_0)\kappa}{(2p_F + \kappa)(\kappa + k_0)}.$$ (11)

where $k_0 = T/v_F$.

Equation (10) derived from the RPA self-energy diagram may be also obtained by the second-order perturbation theory in the electron-electron interaction using the golden rule. Thus we later refer to Eq. (11) as the golden-rule contribution. The coefficient $\pi/4$ in the right-hand-side (rhs) of Eq. (13) coincides with Refs. 10 and 11 and differs from all other published works. References 7 and 9 have $\pi/2$, and Ref. 8 has $\pi/2$. We also note that besides the coefficient, our golden-rule contribution [Eq. (11)] differs from the corresponding equation of Ref. 7 by having only negative logarithmic temperature-dependent terms [the second and the third terms in the rhs of Eq. (11)], while Ref. 7 has some positive terms which lead to overestimation of the electron relaxation.

In the three-dimensional case, RPA is valid for high-electron-density $\kappa \ll p_F$ (see, e.g., Ref. 9). In the two-dimensional case, the fact that the $T^2\ln T$ term in Eq. (11) does not depend on the screening momentum $\kappa$ means that, unlike the three-dimensional case, terms beyond the golden rule in the electron relaxation may be equally important for any value of the ratio $\kappa/p_F$. The first nongolden-rule self-energy, also of second order in $V$, is the second diagram shown in Fig. 1. The corresponding electron-electron relaxation time may be presented in the form

$$\frac{1}{\tau_{c,e}^{(2)}(T,\epsilon)} = \frac{8}{\pi \nu} \omega \left[ N(\omega) + n(\epsilon + \omega) \right]$$

$$\times \int \frac{d^2 p}{(2\pi)^2} \frac{d^2 q}{(2\pi)^2} [V(q)\text{Im} G^A(P) \text{Im} G^A(P + Q)] ^2.$$ (12)

Calculations show that logarithmic terms in $1/\tau_{c,e}^{(2)}$ differ from that of $1/\tau_{c,e}^{(1)}$ by a factor $-\frac{1}{2}$, due to absence of a factor $-2$ associated with the electron loop in $1/\tau_{c,e}^{(1)}$, and there are no nongolden-rule temperature-dependent terms in $1/\tau_{c,e}^{(2)}$. The combined result of both contributions $\tau_{c,e}^{(1)}$ and $\tau_{c,e}^{(2)}$ is

$$\frac{1}{\tau_{c,e}(T)} = \frac{\pi T^2}{8\epsilon_F} \left[ \ln \left( \frac{4\epsilon_F}{T} \right) - \ln \left( \frac{2p_F + \kappa}{\kappa + k_0} \right) \right]$$

$$- \frac{2(2p_F - k_0)\kappa}{(2p_F + \kappa)(\kappa + k_0)},$$ (13)

$$\frac{1}{\tau_{c,e}(0,\epsilon)} = \frac{1}{\tau_{c,e}(0,\epsilon)} = \frac{\epsilon^2}{8\pi \epsilon_F} \left[ \ln \left( \frac{4\epsilon_F}{\epsilon} \right) - \ln \left( \frac{2p_F + \kappa}{\kappa + k_1} \right) \right]$$

$$- \frac{2(2p_F - k_1)\kappa}{(2p_F + \kappa)(\kappa + k_1)}.$$ (14)

where $k_1 = \epsilon/v_F$.

All previous works neglected the nongolden-rule diagram and thus overestimated the result by a factor of 2 [compare Eqs. (11) and (13)]. This is the reason why the coefficient in the right-hand-side of Eq. (14) is half the corresponding coefficient in Refs. 7 and 9.

The temperature- and energy-dependent intralayer scattering rates, Eqs. (13) and (14), respectively, are valid in both the low-density $\kappa \gg p_F$ and in the high-density $\kappa \ll p_F$ limits, as has been checked by numerical evaluation of Eqs. (9) and (12). Thus, the observation that the $T^2\ln T$ term in the electron-relaxation rate is independent of the screening parameter $\kappa$, forces a consideration of nongolden-rule contributions.

**Intralayer and interlayer scattering rate**

The electron lifetime was measured in Ref. 3 in a system of two parallel two-dimensional electron planes. The dis-
tance between the electron planes was \( b = 175 - 350 \) Å. In this situation it is important to take into consideration the interaction between electrons in different planes, which modifies screening of interlayer and intralayer electron-electron interaction and also provides an additional channel of electron relaxation.

The nonscreened Coulomb interaction between electrons in different planes separated by the media with the dielectric constant \( \epsilon_1 \) is

\[
U(q) = \frac{2 \pi e^2}{\epsilon_1} \frac{q}{b} \exp(-qb). \tag{15}
\]

Further we will assume for simplicity that \( \epsilon = \epsilon_1 \).

For two identical layers the screened-interlayer potential \( U \) and screened-intralayer potential \( V \) are described by the equations

\[
U = U^0 + V^0 \Pi U + U^0 \Pi V, \quad V = V^0 + V^0 \Pi V U^0 \Pi U. \tag{16}
\]

The solution of Eq. (16) is

\[
U(q) = \frac{k}{\nu} \exp(-qb), \quad V(q) = \frac{1 + \kappa/q}{\nu + 2k + k^2/q} \exp(-2qb). \tag{17}
\]

Because small momentum transfers are important for the electron-electron relaxation we will assume further that \( qb \ll 1 \) to get

\[
U(q) = \frac{1}{\nu q + 2k(1 + kb)}, \quad V(q) = \frac{\kappa(1 + 2kb)}{\nu q + 2k(1 + kb)}. \tag{18}
\]

Calculating the electron-electron relaxation in a double-well system we note that the screened-intralayer electron-electron potential \( V \) contributes to both golden rule and nongolden-rule diagrams while the interlayer potential \( U \) contributes only to the golden-rule diagram. The result of an analytical calculation using Eq. (18) is

\[
\frac{1}{\tau_{ee}(T)} = \frac{\pi T^2}{32 e_F(1 + kb)^2} \left[ \frac{t_1 + (1 + 2kb)^2 + 2}{\ln \left( \frac{2e_F + pb}{T} \right)} \right] \left[ \frac{p b + 2k b(1 + kb)}{k_0 b + 2k b(1 + kb)} \right]^

\[
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\]

Equation (19) is valid for arbitrary electron density and arbitrary relation between the parameters \( p_F b \) and \( k b \). It is important to keep the low-momentum cutoff \( k_0 \) not only in \( T^2 \ln T \) terms, but also in \( T^2 \) terms to keep \( \tau_{ee} \) positively defined, e.g., for \( T = 20 \) K and \( e_F = 60 \) K, we cannot ignore \( k_0 = 0.3 p_F \). The analytic solution, Eq. (19), is possible only for simplified forms of the potentials, Eq. (18), instead of the exact forms, Eq. (17). Numerical estimates show that this approximation is reasonable.

Using the data reported in Refs. 3 and 4, we presented in Fig. 2 the results of theoretical calculations in a broad temperature range and in Table I experimental and theoretical data for one particular temperature \( T = 0.1 e_F \). From Table I and from Fig. 2 we see that our theoretical calculations [Eq. (19)] disagree with experimental data. The reason for this disagreement is not clear. Probably the tunnel-resonance width of Ref. 3 and the resistance-resonance width of Ref. 4 have a contribution of nonrelaxation origin. Note that “excellent agreement” reported in recent papers \(^{10,11}\) seems too optimistic. In particular, specifics of the electron relaxation in a two-layer system, the interlayer relaxation, and the nongolden-rule contribution, which, as have been shown above, diminishes the result by at least a factor of 2, were ignored.

At very low temperatures, when \( \ln(4e_F / T) \gg 1 \), higher-order terms should be considered. This problem was studied by the \( T \)-matrix approximation for the Hubbard model in Ref. 13 and recently for the Coulomb interaction in Ref. 14. It was shown in Ref. 14 that at very low temperatures when only logarithmic terms are important higher-order terms lead to effectively diminishing the coefficient to the \( T^2 \ln T \) term by another factor of 2 in comparison with Eq. (13).

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<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \epsilon_F ) (meV)</th>
<th>( b ) (Å)</th>
<th>( p_F b )</th>
<th>( k b )</th>
<th>( pb )</th>
<th>( \Gamma(0.1) ) exp.</th>
<th>( \Gamma(0.1) ) theor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. 3</td>
<td>5.7</td>
<td>200</td>
<td>4</td>
<td>1</td>
<td>0.048</td>
<td>0.0073</td>
<td></td>
</tr>
<tr>
<td>Ref. 4</td>
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<td>40</td>
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<td>0.8</td>
<td>1</td>
<td>0.031</td>
<td>0.0077</td>
</tr>
</tbody>
</table>

FIG. 2. Dimensionless electron relaxation rate \( \Gamma = [\tau_{ee}(x) / e_F]^{-1} \), \( x = T / e_F \). Experimental data are from Refs. 3 and 4. Theoretical one-layer golden-rule data are from Eq. (11), one-layer golden-rule and non-golden-rule data are from Eq. (13), and two-layer data are from Eq. (19).


