

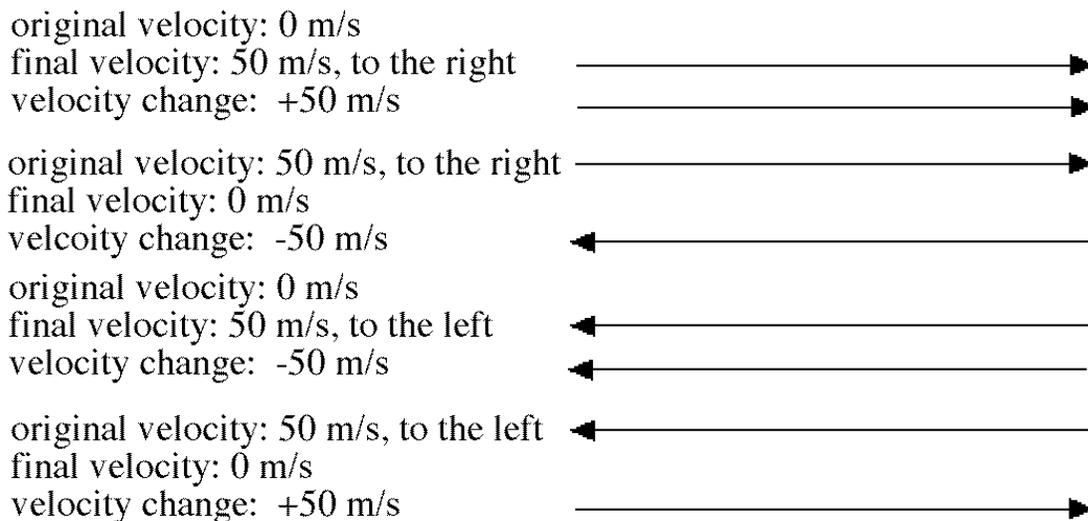
Calculating average acceleration from velocity change and time

Acceleration is a measure of how rapidly the velocity is changing. Since we define average acceleration, a_{av} or a_{av} , as the

$$a_{av} = \frac{\text{change in velocity}}{\text{the time required to change velocity}}$$

the unit of acceleration is (meters per second) per second, (m/s)/s, or m/s^2 . [In the

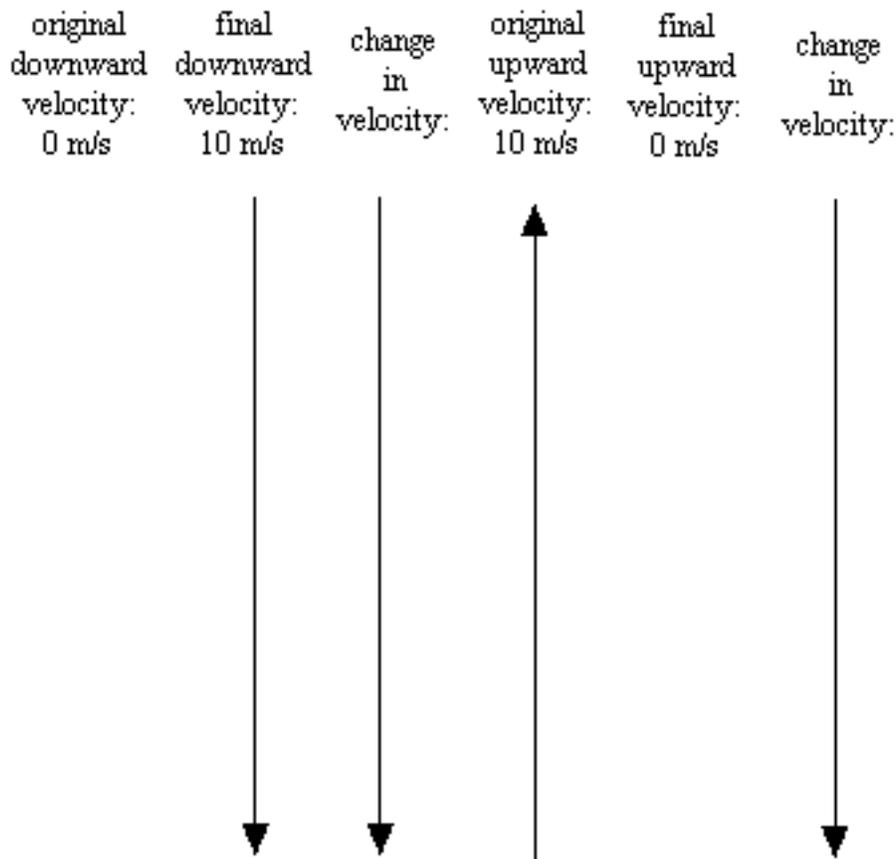
American customary system, the unit of acceleration is the (foot per second) per second, or ft/s^2 .]



Let us suppose that an object changes its velocity from 0 m/s to 50 m/s along some direction in ten seconds, the average acceleration has a magnitude of $50 \text{ m/s}/10 \text{ s} = 5 \text{ m/s}^2$; it has a net average acceleration along the direction specified of $+5 \text{ m/s}^2$. If we suppose instead that an object changes its velocity from 50 m/s to 0 m/s along the same direction in ten seconds, the average acceleration has a magnitude of $50 \text{ m/s}/10 \text{ s} = 5 \text{ m/s}^2$; it has a net average acceleration along the direction specified of -5 m/s^2 . The minus sign indicates that the velocity has decreased along the direction given, in the specified interval. The picture

above shows four possibilities, two with “positive” accelerations (top and bottom), two with “negative” accelerations (middle).

Acceleration may take on both positive and negative values; for this reason, physicists do not usually speak of “deceleration.” Instead, acceleration is given with information allowing the direction of the acceleration to be found. The term “deceleration” merely means that the final speed is less than the initial speed, and is superfluous when the information on initial and final velocities is available. The minus sign is not determined by physics, but by you as you choose the direction of your coordinate axes.



Suppose we look upward and see an object that has zero speed at some instant of time and a speed of 10 m/s downward 1 second later (picture on the left, above). The average

acceleration can be expressed as 10 m/s^2 , downward, or, equivalently, as -10 m/s^2 , upward. Now consider an object that has an initial velocity 10 m/s upward and a final velocity of 0 m/s . It, too, has an acceleration of 10 m/s^2 , downward, or -10 m/s^2 , upward (picture on the right, preceding page). Note from the pictures that the *change in velocity* is the same in each case.

This is roughly the situation that results when any object travels through the air near Earth: it experiences an acceleration of about 10 m/s^2 , downward, or -10 m/s^2 , upward by virtue of its location near Earth's surface. We call the value 9.8 m/s^2 , downward, or -9.8 m/s^2 , upward, **g**, the gravitational acceleration (or acceleration due to gravity). If the object has a large surface area, it will also experience an acceleration due to air resistance, but we shall mostly ignore this complication. The important point here is that *every* object no matter how large or how small will be subject to the *same* gravitational acceleration **g** in Earth's vicinity.

Let's do an example. Consider the speed of an object that you dropped and for which you measure its speed [over a millisecond interval] every second. The results are shown in Table E03.1. What would you find the acceleration to be?

TABLE E03.1

Data for dropped object

time	speed
0.0 s	0 m/s
1.0 s	10.0 m/s
2.0 s	20.0 m/s
3.0 s	29.8 m/s
4.0 s	39.9 m/s
5.0 s	50.1 m/s

The table lists data from 5 seconds worth of time: 6 measurements and 5 intervals. The interval numbers, the time intervals, the measured changes in speed, and the calculated accelerations are shown in Table E03.2. Note that the changes in speed are just about constant. This implies that the acceleration is [just about] constant. It is pretty clear from the data that the acceleration is about 10.0 (m/s)/s. Taking the average, in fact, that is exactly what we get. There are presumably experimental errors made in getting the average speed over the short time interval. After all, at the fifth second, the object travels only 50.1 mm in the 1.00 ms. A slight error in measurement of position of just 0.10 mm could have caused a 0.10 m/s/s error in the acceleration.

TABLE E03.2

Calculating Acceleration using the Data of Table E03.1.

Interval	time interval	change in speed	acceleration
1	1.0 s	10.0 m/s	10.0 m/s/s
2	1.0 s	10.0 m/s	10.0 m/s/s
3	1.0 s	9.8 m/s	9.8 m/s/s
4	1.0 s	10.1 m/s	10.1 m/s/s
5	1.0 s	10.2 m/s	10.2 m/s/s

Suppose an object has an acceleration of 15.0 m/s², west. After 5.0 seconds, what is its speed if it was traveling at 10.0 m/s, east when the clock started ticking?

Since our definition of average velocity is $a_{av} = \frac{\text{change in velocity}}{\text{the time required to change velocity}}$, the change in velocity in a time interval is $a_{av} \times (\text{time interval})$. In this case, the change in velocity is (15.0 m/s², west) \times (5.0 s) = 75.0 m/s, west. Since we began the interval with a velocity of 10.0 m/s, east, the new velocity is the original velocity plus the change in velocity, 10.0 m/s, east + 75.0 m/s, west, or 65.0 m/s, west.

Note on finding acceleration from velocity and time and velocity from position and time

There are relations among position, velocity, and acceleration that can be elucidated in calculus. The instantaneous acceleration (hereafter, simply the acceleration) is the time derivative of the velocity: $a = \frac{dv}{dt}$. The instantaneous velocity (hereafter, simply the velocity) is just the time derivative of the position: $v = \frac{dr}{dt}$. The instantaneousness comes because the process of taking a derivative is simply that of doing a limiting process; in the velocity, for example, it is

$$v = \lim_{t \rightarrow 0} \frac{r \text{ at time } t + \Delta t - r \text{ at time } t}{\Delta t},$$

so the process of taking the limit assures that the value determined is at an instant of time.

Contrariwise, the velocity is the integral (or the antiderivative) of the acceleration. The position is the integral (or the antiderivative) of the velocity. These relations are discussed extensively in physics textbooks that use calculus.