Physics 880.06: Problem Set 5

Note: please turn these problems into the mailbox of the grader, Wissam Al-Saidi, by 5PM Friday, May 16.

1. Consider the Ginzburg-Landau differential equation for ψ as applied to an order parameter ψ which varies in only one spatial direction, say z. If there is no vector potential, this differential equation can be written

$$\alpha\psi + \beta|\psi|^2\psi - \frac{\hbar^2}{2m^*}\psi''(z) = 0, \tag{1}$$

where the primes denote differentiation with respect to z.

(a). Assume that $\alpha < 0$ (as expected for $T < T_c$. Show that one solution of this differential equation is

$$\psi(x) = \psi_0 \tanh(z/z_0), \tag{2}$$

for a suitable choice of ψ_0 and z_0. Also, find ψ_0 and z_0 in terms of the coefficients of the differential equation.

Now we will apply this result to a semi-infinite superconductor occupying the half-space $z > 0$. We imagine that the region $z < 0$ is occupied by some non-superconducting material, and that ψ satisfies the boundary conditions $\psi(z = 0) = 0$, $\psi(z \to \infty) = psi_0$.

The Ginzburg-Landau free energy of the superconductor (per unit area of the boundary at z = 0, and still assuming no vector potential)

$$F_s = \int_0^\infty \left[\alpha|\psi(z)|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\hbar^2}{2m^*}\left|\frac{d\psi}{dz}\right|^2\right] dz. \tag{3}$$

(b). Calculate the *extra* free energy of the superconductor associated with the wall. In other words, calculate the difference between the above free energy and the analogous free energy for a uniform system occupying the half-space $z > 0$. Assume no vector potential and no magnetic fields.

2. Consider a superconducting material which has the property that the transition temperature is a function of position x, with a maximum T_c at $x = 0$. We assume that T_c has the simple parabolic position dependence

$$T_c(x) = T_c(0) - \gamma x^2, \tag{4}$$

where γ is a positive constant. We also assume that the Ginzburg-Landau parameters β and m^* are independent of position, and that the coefficient of the quadratic term, α, has the form

$$\alpha(T, x) = \alpha'(T - T_c(x)), \tag{5}$$

where α' is a positive constant.

In the presence of a vector potential $\mathbf{A}$, the linearized Ginzburg-Landau equation is approximately

$$\alpha(x)\psi + \frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)^2 \psi = 0. \tag{6}$$

(a) Assume a uniform $\mathbf{B}$ field in the z direction, and write down this differential equation explicitly, choosing the Landau gauge for $\mathbf{A}$.

(b). Solve this equation to find the highest temperature $T_c(B)$ for which there is a solution. How does this solution depend on the parameter γ?