Physics 880.06: Problem Set 4

Note: please turn these problems into the mailbox of the grader, Wissam Al-Saidi, by 12 noon on Monday, May 5. This problem set consists of just a single problem, which is worth 25 points.

1. The BCS equation for the gap function $\Delta_{\mathbf{k}}$ at temperature $T = 0$ is

$$\Delta_{\mathbf{k}} = -\frac{1}{2}\sum_{\mathbf{k}'}\frac{\Delta_{\mathbf{k}'}V_{\mathbf{k},\mathbf{k}'}}{\sqrt{\Delta_{\mathbf{k}'}^2+\xi_{\mathbf{k}'}^2}}, \tag{1}$$

where $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - E_F$, and $\epsilon_{\mathbf{k}}$ is the energy of a non-interacting electron of wave vector $\mathbf{k}$.

(a). Show that if $V_{\mathbf{k},\mathbf{k}'} = -A_{\mathbf{k}}A_{\mathbf{k}'}$, where $A_{\mathbf{k}}$ is a specified function of $\mathbf{k}$, then the above equation is solved by a gap function of the form $\Delta_{\mathbf{k}} = A_{\mathbf{k}}\Delta$, where Δ is independent of $\mathbf{k}$, and find an equation for Δ.

(b). Now suppose specifically that our superconductor is two-dimensional, and that $A_{\mathbf{k}}$ has the form

$$A_{\mathbf{k}} = A_0\cos(2\phi) \tag{2}$$

if $|\epsilon_{\mathbf{k}} - E_F| < \hbar\omega_c$, and

$$A_{\mathbf{k}} = 0 \tag{3}$$

otherwise. Here $\hbar\omega_c$ is a cutoff energy, $\epsilon_{\mathbf{k}} = \hbar^2k^2/(2m)$, and we have written $\mathbf{k} = (k\cos\phi, k\sin\phi)$. Obtain an integral expression which determines Δ in this case, but you need not solve this to obtain Δ explicitly.

(c). The elementary excitations ("Bogoliubons") described in class are Fermions which have the dispersion relation $E_{\mathbf{k}} = \sqrt{[\Delta A_{\mathbf{k}}]^2 + \xi_{\mathbf{k}}^2}$. Show that these Bogoliubons are gapless - that is, they have vanishing energies at certain specific values of $\mathbf{k}$ (NOT at all values of $\mathbf{k}$!). What are those values of $\mathbf{k}$?

(d). Let $\mathbf{k}_0$ be a node point for a Bogoliubon, and let $\delta\mathbf{k} = \mathbf{k} - \mathbf{k}_0$. Show that, if one chooses the coordinate axes of the vector $\delta\mathbf{k}$ suitably, the energies of the Bogoliubons sufficiently near the point $\mathbf{k}_0$ have the *approximate* form

$$E(\delta\mathbf{k}) \sim \sqrt{C_1[\delta k_1]^2 + C_2[\delta k_2]^2} \tag{4}$$

where δk_1 and δk_2 are the components of $\delta \mathbf{k}$ in the new coordinate system, and C_1 and C_2 are positive constants.

(e). What is the density of states of these Bogoliubons at very low energy?

(f). If we neglect the temperature-dependence of the gap variable Δ, what is the functional form of the temperature-dependence of the specific heat at very low temperatures, for a superconductor with this type of energy gap?

Note: This type of gap function is believed to describe some of the cuprate-based high-temperature superconductors.