

Physics 880.06: Problem Set 6

Note: please turn these problems into the mailbox of the grader, Wissam Al-Saidi, by the beginning of class on Thursday, May 23.

1. Consider a single Abrikosov vortex parallel to the z axis. Assume that this vortex experiences three forces. The first is a “Magnus” force due to an applied uniform ac current density, $\mathbf{J}_{ac}(t) = J_{ac}\hat{x} \cos(\omega t)$. The second is a frictional force $\mathbf{F}_f = -\eta\mathbf{v}$, where \mathbf{v} is the velocity. The last is a “pinning” force $F_{pin} = -k\mathbf{r}$ per unit vortex length, where $\mathbf{r} = (x, y)$ is the two dimensional vector describing the position of the vortex in the xy plane and k is a pinning constant.
 - (a). Find the vortex velocity $\mathbf{v}(t)$. (Consider only the vortex velocity induced by the applied current \mathbf{J} .)
 - (b). Suppose the current density is instead $J_{ac}(\hat{x} \cos(\omega t) + \hat{y} \sin(\omega t))$. Find the vortex velocity in this case.
 - (c). Describe the vortex orbit in each case.
 - (d). Would either of the motions (a) or (b) produce a dc electric field? An ac electric field?
2. In class, I wrote down, without much of an argument, the relation $E = vB/c$, where v is the velocity of a moving Abrikosov lattice, B is the average magnetic induction, and E is the induced electric field. Show that this result can be obtained from the Josephson relation between voltage and phase as follows. Consider a line of length L perpendicular to the velocity of the Abrikosov lattice. Calculate the number of vortices crossing this line per second. Calculate how much the phase difference between the two ends of the line changes each time a vortex crosses the line. Then use the Josephson relation to get the voltage drop across the line. Divide by the length to get the electric field. You should get the same result as at the beginning of the problem. What is the direction of the E-field, relative to v and B ?
3. Show explicitly that the differential equation for the gauge-invariant phase in the RCSJ model is mathematically equivalent to the equation of motion for a damped pendulum in a gravitational field and subject

to an additional driving force. Find an explicit mapping between the parameters of the RCSJ model and those of the pendulum. (Note: do *not* consider the pendulum in the small angle approximation.)

4. Make a numerical estimate of the parameters in the RCSJ model for a typical SIS junction. Assume that the S elements are aluminum (with a gap of about 5K), assume the Ambegaokar- Baratoff form for the critical current, assume a junction area of $[2000 \text{ \AA}]^2$, a junction spacing of about 100 \AA , and a shunt resistance of 4000Ω . Make a reasonable estimate for the capacitance. Is this junction overdamped or underdamped? Estimate the junction quality factor Q , and the plasma frequency ω_p .