

### Physics 880.06: Problem Set 5

Note: please turn these problems into the mailbox of the grader, Wissam Al-Saidi, by the beginning of class on Tuesday, May 14.

1. Consider the Ginzburg-Landau differential equation for  $\psi$  as applied to an order parameter  $\psi$  which varies in only one spatial direction, say  $z$ . If there is no vector potential, this differential equation can be written

$$\alpha\psi + \beta|\psi|^2\psi - \frac{\hbar^2}{2m^*}\psi''(z) = 0, \quad (1)$$

where the primes denote differentiation with respect to  $z$ .

- (a). Assume that  $\alpha < 0$  (as expected for  $T < T_c$ ). Show that one solution of this differential equation is

$$\psi(x) = \psi_0 \tanh(z/z_0), \quad (2)$$

for a suitable choice of  $\psi_0$  and  $z_0$ . Also, find  $\psi_0$  and  $z_0$  in terms of the coefficients of the differential equation.

Now we will apply this result to a semi-infinite superconductor occupying the half-space  $z > 0$ . We imagine that the region  $z < 0$  is occupied by some non-superconducting material, and that  $\psi$  satisfies the boundary conditions  $\psi(z = 0) = 0$ ,  $\psi(z \rightarrow \infty) = \psi_0$ .

The Ginzburg-Landau free energy of the superconductor (per unit area of the boundary at  $z = 0$ , and still assuming no vector potential)

$$F_s = \int_0^\infty \left[ \alpha|\psi(z)|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\hbar^2}{2m^*} \left| \frac{d\psi}{dz} \right|^2 \right] dz. \quad (3)$$

- (b). Calculate the *extra* free energy of the superconductor associated with the wall. In other words, calculate the difference between the above free energy and the analogous free energy for a uniform system occupying the half-space  $z > 0$ . Assume no vector potential and no magnetic fields.

2. Consider a superconducting material which has the property that the transition temperature is a function of position  $x$ , with a maximum  $T_c$  at  $x = 0$ . We assume that  $T_c$  has the simple parabolic position dependence

$$T_c(x) = T_c(0) - \gamma x^2, \quad (4)$$

where  $\gamma$  is a positive constant. We also assume that the Ginzburg-Landau parameters  $\beta$  and  $m^*$  are independent of position, and that the coefficient of the quadratic term,  $\alpha$ , has the form

$$\alpha(T, x) = \alpha'(T - T_c(x)), \quad (5)$$

where  $\alpha'$  is a positive constant.

In the presence of a vector potential  $\mathbf{A}$ , the linearized Ginzburg-Landau equation is approximately

$$\alpha(x)\psi + \frac{1}{2m^*} \left( -i\hbar\nabla - \frac{e^*}{c}\mathbf{A} \right)^2 \psi = 0. \quad (6)$$

- (a) Assume a uniform  $\mathbf{B}$  field in the  $z$  direction, and write down this differential equation explicitly, choosing the Landau gauge for  $\mathbf{A}$ .
- (b). Solve this equation to find the highest temperature  $T_c(B)$  for which there is a solution. How does this solution depend on the parameter  $\gamma$ ?