

## Physics 880.06: Problem Set 9

Due Friday, December 4, 2003

Note: each problem is worth 10 pts.

1. *Magnetoresistance in a two-band model.* It was shown in class that, for electrons in a spherical band minimum, the current density  $\mathbf{J}$  and electric field  $\mathbf{E}$  were related by the equation

$$E_i = \sum_{j=1}^3 \rho_{ij} J_j, \quad (1)$$

where  $E_i$  and  $J_i$  are the  $i^{\text{th}}$  Cartesian components of  $\mathbf{E}$  and  $\mathbf{J}$ , and the resistivity tensor  $\rho_{ij}$  has components

$$\rho_{xx} = \rho_{yy} = \rho_{zz} = \frac{1}{\sigma_0} \quad (2)$$

$$\rho_{xy} = -\rho_{yx} = -R_H B, \quad (3)$$

with all other components vanishing. Here  $B$  is the magnetic field (assumed along the  $z$  direction),  $n_e$  is the electron density,  $\sigma_0 = n_e e^2 \tau / m_e$ ,  $R_H = -1/(n_e e c)$  is the Hall coefficient,  $\tau$  is the relaxation time, and  $m_e$  is the effective mass.

(a). Find the conductivity tensor  $\sigma$ , which is the matrix inverse of the resistivity tensor.

(b). Suppose we have two bands, denoted 1 and 2, which both have spherical band minima and carry current in parallel. The corresponding conductivity tensor is  $\sigma_1 + \sigma_2$ , where  $\sigma_1$  and  $\sigma_2$  are just the analogs of the conductivity tensor  $\sigma$  found in (a), but with different electron densities  $n_\alpha$ , relaxation times  $\tau_\alpha$ , and effective masses  $m_\alpha$  ( $\alpha = 1, 2$ ).

Calculate the resistivity tensor in this two-band case. Show that in this case  $\rho_{xx}$  depends on  $B$  but saturates (approaches a constant) at high fields (i. e. the transverse magnetoresistance is finite). Also, find the Hall coefficient  $R_H = -\rho_{xy}/B$ ; show that it, too, depends on magnetic field but approaches a constant at high fields, and find that constant in terms of the parameters of the two bands.

2. Repeat the previous problem, but for an electron and a hole band. What happens if the electron and hole densities are equal?
3. Consider a parabolic band minimum in which the electronic energies are described by the relation

$$E(\mathbf{k}) = E_0 + \sum_{i=1}^3 \frac{\hbar^2 k_i^2}{2m_i}, \quad (4)$$

where  $m_i$  is the  $i^{\text{th}}$  principal component of the effective mass tensor. Let the electron density be  $n_e$ . Derive an expression for the Fermi energy  $E_F$  in terms of  $n_e$  and the  $m_i$ 's. Show that your result reduces to the usual one when all the  $m_i$ 's are equal.

4. Consider a single-band conductor in which the band energies are described by eq. (4). Solve the Boltzmann equation in the relaxation time approximation, as was done in class, for this anisotropic case, and show that at low temperatures the conductivity tensor takes the form

$$\sigma_{ij} = \frac{ne^2\tau}{m_i} \delta_{ij}, \quad (5)$$

where  $\delta_{ij}$  is the Kronecker delta function:  $\delta_{ij} = 1$  if  $i = j$  and 0 otherwise.  $n$  is the electron density.