Physics 880.06: Problem Set 6

Due Friday, November 7, 2003

1. Ashcroft and Mermin, Chapter 9, Problem 3.

2. Consider the tight-binding approximation for an s-band in a triangular lattice with lattice constant a. Let the normalized atomic wave function be denoted $\phi(\mathbf{r}$. Assume that the hopping matrix element $H_{\mathbf{R},\mathbf{R}'} \equiv \int_d^3 r\phi(\mathbf{r}-\mathbf{R})H\phi(\mathbf{r}-\mathbf{R}') = -t$ $(t > 0)$ if $\mathbf{R}$ and $\mathbf{R}'$ are nearest neighbor Bravais lattice vectors, $H_{\mathbf{R},\mathbf{R}'} = \epsilon_0$ if $\mathbf{R} = \mathbf{R}'$, and $H_{\mathbf{R},\mathbf{R}'} = 0$ otherwise.

   (a) Find the band energies $E(\mathbf{k})$ in terms of t, a, and ϵ_0.

   (b). Show that near the bottom of the band $E(\mathbf{k}) = E_{min}+\hbar^2k^2/(2m^*)$, and find E_{min} and m^* in terms of ϵ_0, t, and a.

   (c). What is the band width, in terms of t, ϵ_0, and a?

3. *For edification only; not to be turned in.* Consider a layered dielectric consisting of alternate layers of thickness d_1 and d_2, having dielectric constants ϵ_1 and ϵ_2, and consider a linearly polarized plane electromagnetic wave propagating perpendicular to the layers. Let this be denoted the z direction.

   (a). Write down the form of the electric field, given that it must satisfy Bloch's theorem. Write down the corresponding magnetic field $\mathbf{B}$. (Assume that the relative permeability $\mu = 1$ in both media.)

   (b). What are the boundary conditions on $\mathbf{E}$ and $\mathbf{B}$ at $z = d_1$ and $z = d_1 + d_2$?

   (c). Using (a) and (b), find a determinantal equation which gives a relationship between the Bloch vector k and the frequency ω. Don't try to solve this equation.