Physics 880.06: Problem Set 2

Due Friday, October 10, 2003

1. *Free electron gas in two dimensions.*

   (a). Find the relation between n and k_F in 2d.

   (b). Calculate the electronic density of states per unit volume, g(E), in 2d. Show, in particular, that g(E) is independent of energy for $E > 0$ and g(E) = 0 for $E < 0$.

   (c). Using the fact that the chemical potential is determined by the equation

   $$\int_{-\infty}^{\infty} dE g(E) f(E) = n, \tag{1}$$

   where n is the electron density, show that $\mu(T)$ is given implicitly by

   $$\mu + k_B T \ln(1 + e^{-\mu/k_B T}) = E_F, \tag{2}$$

   where E_F is the Fermi energy.

2. Ashcroft and Mermin, Chapter 2, Problem 2, part (a) only.

3. Consider the density of states g(E) per unit volume for spin-1/2 fermions in d dimensions (d = 1, 2, or 3), on the assumption that the relation between E and k is

   $$E = Ak^n. \tag{3}$$

   Specifically, show that $g(E) \propto E^\alpha$, and find the power α as a function of n and d. (For example, in three dimensions, for n= 2, we showed in class that $\alpha = 1/2$.)

Note: since I will be out of town on the due date, please turn in the set by 5PM in the box of the grader, Mr. Wissam Al-Saidi. Thank you.