Physics 880.06: Problem Set 9

Due Friday, December 5, 2002

Note: each problem is worth 10 pts.

1. *Magnetoresistance in a two-band model.* It was shown in class that, for electrons in a spherical band minimum, the current density $\mathbf{J}$ and electric field $\mathbf{E}$ were related by the equation

$$E_i = \sum_{j=1}^{3} \rho_{ij} J_j, \tag{1}$$

where E_i and J_i are the ith Cartesian components of $\mathbf{E}$ and $\mathbf{J}$, and the resistivity tensor ρ_{ij} has components

$$\rho_{xx} = \rho_{yy} = \rho_{zz} = \frac{1}{\sigma_0} \tag{2}$$

$$\rho_{xy} = -\rho_{yx} = -R_H B, \tag{3}$$

with all other components vanishing. Here B is the magnetic field (assumed along the z direction), n_e is the electron density, $\sigma_0 = n_e e^2 \tau / m_e$, $R_H = -1/(n_e e c)$ is the Hall coefficient, τ is the relaxation time, and m_e is the effective mass.

(a). Find the conductivity tensor σ, which is the matrix inverse of the resistivity tensor.

(b). Suppose we have two bands, denoted 1 and 2, which both have spherical band minima and carry current in parallel. The corresponding conductivity tensor is $\sigma_1 + \sigma_2$, where σ_1 and σ_2 are just the analogs of the conductivity tensor σ found in (a), but with different electron densities n_α, relaxation times τ_α, and effective masses m_α ($\alpha = 1, 2$).

Calculate the resistivity tensor in this two-band case. Show that in this case ρ_{xx} depends on B but saturates (approaches a constant) at high fields (i. e. the transverse magnetoresistance is finite). Also, find the Hall coefficient $R_H = -\rho_{xy}/B$; show that it, too, depends on magnetic field but approaches a constant at high fields, and find that constant in terms of the parameters of the two bands.

2. Repeat the previous problem, but for an electron and a hole band. What happens if the electron and hole densities are equal?

3. Consider a parabolic band minimum in which the electronic energies are described by the relation

$$E(\mathbf{k}) = E_0 + \sum_{i=1}^{3} \frac{\hbar^2 k_i^2}{2m_i}, \tag{4}$$

where m_i is the ith principal component of the effective mass tensor. Let the electron density be n_e. Derive an expression for the Fermi energy E_F in terms of n_e and the m_i's. Show that your result reduces to the usual one when all the m_i's are equal.

4. Consider a single-band conductor in which the band energies are described by eq. (4). Solve the Boltzmann equation in the relaxation time approximation, as was done in class, for this anisotropic case, and show that at low temperatures the conductivity tensor takes the form

$$\sigma_{ij} = \frac{ne^2\tau}{m_i}\delta_{ij}, \tag{5}$$

where δ_{ij} is the Kronecker delta function: $\delta_{ij} = 1$ if $i = j$ and 0 otherwise. n is the electron density.