1. Ashcroft and Mermin, Chapter 9, Problem 3.

2. Consider the tight-binding approximation for an $s$-band in a triangular lattice with lattice constant $a$. Let the normalized atomic wave function be denoted $\phi(\mathbf{r})$. Assume that the hopping matrix element $H_{\mathbf{R},\mathbf{R}'} \equiv \int_{\mathbb{R}^3} d\mathbf{r} \phi(\mathbf{r} - \mathbf{R}) H \phi(\mathbf{r} - \mathbf{R}') = -t$ ($t > 0$) if $\mathbf{R}$ and $\mathbf{R}'$ are nearest neighbor Bravais lattice vectors, $H_{\mathbf{R},\mathbf{R}'} = \epsilon_0$ if $\mathbf{R} = \mathbf{R}'$, and $H_{\mathbf{R},\mathbf{R}'} = 0$ otherwise.

(a) Find the band energies $E(k)$ in terms of $t$, $a$, and $\epsilon_0$.

(b). Show that near the bottom of the band $E(k) = E_{\text{min}} + \hbar^2 k^2 / (2m^*)$, and find $E_{\text{min}}$ and $m^*$ in terms of $\epsilon_0$, $t$, and $a$.

(c). What is the band width, in terms of $t$, $\epsilon_0$, and $a$?

3. For edification only; not to be turned in. Consider a layered dielectric consisting of alternate layers of thickness $d_1$ and $d_2$, having dielectric constants $\epsilon_1$ and $\epsilon_2$, and consider a linearly polarized plane electromagnetic wave propagating perpendicular to the layers. Let this be denoted the $z$ direction.

(a). Write down the form of the electric field, given that it must satisfy Bloch’s theorem. Write down the corresponding magnetic field $\mathbf{B}$. (Assume that the relative permeability $\mu = 1$ in both media.)

(b). What are the boundary conditions on $\mathbf{E}$ and $\mathbf{B}$ at $z = d_1$ and $z = d_1 + d_2$?

(c). Using (a) and (b), find a determinantal equation which gives a relationship between the Bloch vector $k$ and the frequency $\omega$. Don’t try to solve this equation.