Physics 880.06: Problem Set 1; Due Thursday, April 7, 2011 at 11:59 P. M.

1. (20) *Thermodynamics of the Superconducting State:* The equilibrium state of a superconductor in a uniform magnetic field is determined by the temperature T and magnitude of the field H. (Assume that the pressure P is fixed and that the superconductor is a long cylinder parallel to the field, so that demagnetization effects are negligible.) We write the differential relation for the Gibbs free energy density as

$$dG = -SdT - MdH, \tag{1}$$

where G is the Gibbs free energy per unit volume, S is the entropy per unit volume, and M is the magnetization. The phase boundary between the S and N states in a magnetic field is defined by $H_c(T)$ (for a Type I superconductor).

(a). Show, from the fact that G is continuous across the phase boundary, that

$$\frac{dH_c}{dT} = -\frac{S_n - S_s}{M_n - M_s}. \tag{2}$$

(b). Using the fact that the superconducting state displays perfect diamagnetism (B = 0) while the normal state has $M \sim 0$, show that

$$S_n - S_s = -\frac{1}{4\pi}H_c\frac{dH_c}{dT}. \tag{3}$$

and hence the latent heat per unit volume, when the transition occurs in a field, is

$$Q = -T\frac{H_c}{4\pi}\frac{dH_c}{dT}. \tag{4}$$

(c). Show that when the transition occurs at zero field, there is a specific heat discontinuity given by

$$C_N - C_S = -\frac{T}{4\pi}\left(\frac{dH_c}{dT}\right)^2. \tag{5}$$

2. (20) Consider an infinite superconducting slab bounded by two parallel planes perpendicular to the y-axis at y = ± d. Let a uniform magnetic field of strength H_0 be applied along the z axis.

   (a) Taking as a boundary condition that the parallel component of **B** be continuous at the surface, deduce from the appropriate London equation and Ampere's Law that

$$\mathbf{B} = B(y)\hat{z}, \tag{6}$$

   where

$$B(y) = H_0 \frac{\cosh(y/\lambda)}{\cosh(d/\lambda)}. \tag{7}$$

   (b). Show that the diamagnetic current flowing in equilibrium is

$$\mathbf{J} = J(y)\hat{x} \tag{8}$$

   where

$$J(y) = \frac{c}{4\pi\lambda} H_0 \frac{\sinh(y/\lambda}{\cosh(d/\lambda)}. \tag{9}$$

   (c). The magnetization density at a point within the slab is $\mathbf{M}(y) = (\mathbf{B}(y) - \mathbf{H}_0)/(4\pi)$. Show that the magnetization density (averaged over the thickness of a slab) is

$$\bar{M} = -\frac{H_0}{4\pi}\left(1 - \frac{\lambda}{d}\tanh\frac{d}{\lambda}\right), \tag{10}$$

   and give the limiting form of the susceptibility when the slab is very thick or very thin.

3. (10) Consider a superconducting sphere in an external static magnetic field below the critical field. Solve the magnetostatic problem to find the magnetic field distribution outside the sphere, assuming zero penetration depth. Also calculate the magnetic moment of the sphere under these circumstances.