Physics 836: Problem Set 7

Due Wednesday, June 1 by 5PM

Note: since the final is on Friday, June 3, I can't accept this set late. Thanks
All problems worth 10 pts. unless otherwise stated.

1. A superconducting slab occupies the region $-d/2 < z < d/2$. The region outside this slab is non-superconductor. A magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{x}}$ is applied to the superconductor. Thus, the field is $\mathbf{B}_0$ just outside the slab on either side.

   (a). Using the London equations, find the magnetic field everywhere inside the slab, in terms of the London penetration depth λ_L and thickness d.

   (b) Find the current density $\mathbf{J}$ everywhere inside the slab.

2. Consider a superconducting sphere of radius a in an applied magnetic field $\mathbf{H}$. Suppose that the penetration depth $\lambda \ll a$, so that the magnetic field can be regarded as excluded from the sphere.

   (a). Calculate the $\mathbf{B}$ field outside the sphere. Hint: use the magnetic scalar potential and also use Gaussian units.

   (b). Find the field at the equator just outside the sphere.

   (c). Calculate the induced surface current density, and hence the induced magnetic moment.

3. A common approximation to treat the ac properties of superconductors is the *two-fluid model*. In this model, it is assumed that there are two types of charge carriers. One is a superconducting carrier, which has charge 2e and mass $2m_e$. The normal carrier has charge e and mass m_e. The superconducting carriers satisfy the equation of motion

$$2m_e\frac{d\mathbf{v}_s}{dt} = 2e\mathbf{E}, \tag{1}$$

   and the normal carriers satisfy

$$m_e\frac{d\mathbf{v}_n}{dt} = e\mathbf{E} - \frac{m_e}{\tau}\mathbf{v}_n, \tag{2}$$

where $\mathbf{E}$ is an applied electric field and τ is a characteristic relaxation time for normal carriers. We assume also that the number density of superconducting carriers is n_s and that of normal carriers is n_n. The total current density is the sum of the current densities of superconducting and normal carriers.

Suppose that the applied electric field is an ac field of frequency ω, so that $\mathbf{E} = \mathbf{E}_0 \exp(-i\omega t)$.

(a). Find an expression for the total current density $\mathbf{J} = \mathbf{J}(\omega) \exp(-i\omega t)$.

(b). Hence, find an expression for the total conductivity $\sigma(\omega)$. (This will be a complex quantity.) Show that the conductivity is purely imaginary and diverges at very low frequency.

4. In class, I derived the "effective medium approximation" for a two-component material having a volume fraction f_1 of material 1 and $f_2 = 1 - f_1$ of material 2. The two components were assumed to have conductivities σ_1 and σ_2, respectively. The resulting equation determining the effective conductivity σ_e was

$$f_1 \frac{\sigma_1 - \sigma_e}{\sigma_1 + 2\sigma_e} + (1 - f_1) \frac{\sigma_2 - \sigma_e}{\sigma_2 + 2\sigma_e} = 0. \tag{3}$$

This derivation assumed that the grains of each type of material were approximately spherical, and that the medium was three-dimensional.

In this problem, you will consider the same type of material in *two* dimensions. Suppose that we have an *area* fraction f_1 of conductor 1, with conductivity σ_1, and $f_2 = 1 - f_1$ of conductor 2, of conductivity σ_2.

(a). Assume that the "grains" of each type of material are approximately *circular*, and find the equation determining the effective conductivity σ_e in two dimensions.

(b). Obtain the limiting behavior when (i) $\sigma_2 = 0$, and (ii) $\sigma_2 = \infty$, and interpret your results.