Physics 836: Problem Set 6

Due Wednesday, May 25 by 5PM

All problems worth 10 pts. unless otherwise stated.

1. Jackson, problem 13.10 (a), (c).

2. (a) Consider the reciprocal lattice of a Bravais lattice. Show that the reciprocal of this reciprocal lattice is the original Bravais lattice.

   (b). Show that the reciprocal of a face-centered cubic lattice is a body-centered-cubic lattice.

   (c). A *simple hexagonal* lattice has Bravais lattice vectors $\mathbf{a}_1 = a\hat{x}$, $\mathbf{a}_2 = \frac{a}{2}\hat{x} + \frac{\sqrt{3}a}{2}\hat{y}$, $\mathbf{a}_3 = c\hat{z}$, where a and c are constants.

   Show that the reciprocal of this simple hexagonal lattice is also a simple hexagonal lattice, but rotated around the z axis relative to the original one.

3. Consider a periodic layered *magnetically permeable* medium, with alternating layers of permeability μ_1 and μ_2 and thicknesses d_1 and d_2. The dielectric constant of each layer is unity. (We are using esu here.) Consider a linearly polarized plane electromagnetic wave of frequency ω propagating in a direction perpendicular to the layers. The first layer (of medium 1) occupies the region $0 < z < d_1$, and the second (of medium 2) occupies the region $d_1 < z < d_1 + d_2$.

   (a) Write down the most general form of the wave (i) for $0 < z < d_1$; (ii) for $d_1 < z < d_1 + d_2$.

   (b). Use Bloch's theorem, combined with the known boundary conditions at d_1 and $d_1 + d_2$, to write down four conditions on the four unknown amplitudes you should have found in part (a).

   (c). Hence, write down a determinantal condition which determines the frequencies for a given Bloch vector k. Do not attempt to solve to actually find these frequencies, however.