

## Physics 836: Problem Set 5

Instructor: D. Stroud

Due Wednesday, May 18 by 5PM

All problems worth 10 pts. unless otherwise stated.

1. Fill in the steps needed to go from Jackson, eq. (14.25) to Jackson, eq. (14.26)
2. Verify eq. (14.66)
3. Jackson (14.9) (a), (b), (c).
4. Jackson (14.12).
5. OPTIONAL; not to be turned in. (But recommended.)

In class, we have discussed the *inhomogeneous wave equation*

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \Phi(\mathbf{x}, t) = -4\pi\rho(\mathbf{x}, t), \quad (1)$$

where  $\Phi(\mathbf{x}, t)$  is the potential and  $\rho(\mathbf{x}, t)$  is the charge density. We used the fact, derived last quarter, that a particular solution to this differential equation is

$$\Phi(\mathbf{x}, t) = \int G^{ret}(\mathbf{x}, t; \mathbf{x}', t') \rho(\mathbf{x}', t') d^3x' dt', \quad (2)$$

where

$$G^{ret}(\mathbf{x}, t; \mathbf{x}', t') = \frac{\delta(t' - [t - |\mathbf{x} - \mathbf{x}'|/c])}{|\mathbf{x} - \mathbf{x}'|} \quad (3)$$

is the *retarded Green's function* for the wave equation, which satisfies

$$\left(\nabla_x^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) G^{ret}(\mathbf{x}, t; \mathbf{x}', t') = -4\pi\delta(\mathbf{x} - \mathbf{x}')\delta(t - t'). \quad (4)$$

Eq. (3) is obtained by Fourier transforming  $G^{ret}(R, \omega)$ , as follows:

$$G^{ret}(R, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{ikR}}{R} e^{-i\omega\tau} d\omega, \quad (5)$$

where  $k = \omega/c$ ,  $R = |\mathbf{x} - \mathbf{x}'|$  and  $\tau = t - t'$ . This expression for  $G^{ret}(R, \omega)$  is derived in Jackson, eqs. (6.38) - (6.40) and was also derived last quarter in class.

(a). Evaluate this integral by using a suitable contour integral (it will be in either the upper or lower half  $\omega$  plane) that this expression reduces to eq. (3).

(b). Repeat (a), but using a suitable contour integral for the *advanced* Green's function

$$G^{adv}(R, \omega) = \frac{e^{-ikR}}{R}, \quad (6)$$

to show that one gets eq. (6.44) for  $G^{adv}(R, \tau)$ .