Physics 836: Problem Set 4

Due Wednesday, May 11 by 5PM

All problems worth 10 pts. unless otherwise stated.

1. Jackson 11.21(a).

2. As we showed in class, in a large electromagnetic cavity of volume V = L^3, the allowed k points are (k_x, k_y, k_z), where $k_x = 2\pi n_x/L$, $k_y = 2\pi n_y/L$, and $k_z = 2\pi n_z/L$, and n_x, n_y, and n_z are integers.

   (a). Find the density of modes per unit volume in k-space, recalling that there are two modes for each k point. (This was almost done in class.)

   (b). Calculate the number of modes between angular frequency ω and $\omega + d\omega$, in the limit of very large L.

   (c). Suppose that the total energy in a mode of frequency ω at temperature T is $\hbar\omega/[exp(\hbar\omega/k_B T) - 1]$, where $\hbar$ is Planck's constant. Let $E(\omega)d\omega$ equal the energy in the cavity between frequency ω and $\omega + d\omega$. Find the frequency for which $E(\omega)$ is maximum at a given temperature. Estimate this frequency for (a) room temperature, and (b) the surface temperature of the sun.

3. Show that the Euler-Lagrange equations, applied to the Lagrangian density $\mathcal{L}$ given in eq. (12.85), are equivalent to the two inhomogeneous Maxwell equations. (This was already done in class for Coulomb's Law.)

4. Jackson, problem 12.5 (b). [This was previously suggested as an optional problem in the last set.]

5. Jackson, problem 14.5

6. Jackson, problem 14.4. Omit sketch. Here "discuss" means "calculate".