

Physics 835: Problem Set 7.

Due Wednesday, March 2 by 5PM.

1. Prove the following properties of vector spherical harmonics and the operators \mathbf{L} :
 - (a). Jackson (9.20). (This proof was sketched in class.)
 - (b). $\mathbf{L} \times \mathbf{L} = i\mathbf{L}$.
 - (c). Show that the operator multiplying $Y_{\ell m}$ on the left-hand side of eq. (9.99) can be written as $\mathbf{L} \cdot \mathbf{L}$, where \mathbf{L} is defined in eq. (9.101).
2. A radiating source consists of an electric dipole $\mathbf{p} = p \exp(-i\omega t)\hat{x}$ and magnetic dipole $\mathbf{m} = m \exp(-i\omega t)\hat{y}$. Both electric and magnetic dipoles are small compared to the wavelength and both are located at the origin. Find the angular distribution of the time-averaged radiated power per unit solid angle.
3. For the spherical cavity discussed in class, find the frequency of the lowest TE mode, in terms of the radius R of the cavity, and find the corresponding electric and magnetic fields.