Physics 829: Optional Part of Problem Set 3 (not to be turned in)

1. (a) Show that $(\sigma \cdot \mathbf{P})^2 = P^2$, where σ is the ordered triple of Pauli matrices and $\mathbf{P}$ is the momentum operator.

   (b) Show that $\{\alpha_i, \alpha_j\} = 1$ (for $i \neq j$), $\{\alpha_i, \beta\} = 1$, and $\alpha_i^2 = \beta^2 = I$, where $\{,\}$ represents an anticommutator, the α_i and β are the 4×4 matrices defined in Shankar, eq. (20.1.12), and I is the 4×4 unit matrix.

   (c) Show that

   $$(\sigma \cdot \mathbf{A})(\sigma \cdot \mathbf{B}) = i\sigma \cdot \mathbf{A} \times \mathbf{B} + \mathbf{A} \cdot \mathbf{B}, \tag{1}$$

   where $\mathbf{A}$ and $\mathbf{B}$ are any three-component vectors which commute with each other and with the components of σ.

   (d). Show that $\mathbf{\Pi} \times \mathbf{\Pi} = \frac{i\hbar q}{c}\mathbf{B}$, where the operator $\mathbf{\Pi} = \mathbf{P} - q\mathbf{A}/c$, $\mathbf{P}$ is the canonical momentum operator, q is the charge, and $\mathbf{A}$ is the vector potential.

2. Show that the most general Hermitean 2×2 matrix may be written

   $$C_1 I + C_2\sigma_x + C_3\sigma_y + C_4\sigma_z \tag{2}$$

   and thus no Hermitean 2×2 matrix anticommutes with all three of the Pauli matrices except for the 2×2 matrix $\mathbf{0}$ all of whose components are zero.