Some Corrections to Notes 2

P. 21 and the first two lines of p. 22 of the notes should be replaced by the following.

Now the most general solution in the region where there is no optential can be written

$$\sum_{\ell=0}^{\infty} \left[D_\ell j_\ell(kr) + F_\ell n_\ell(kr)\right] P_\ell(\cos\theta)$$

which asymptotically takes the form (for large kr)

$$\sum_{\ell=0}^{\infty} \frac{1}{kr} \left[D_\ell \sin(kr - \ell\pi/2) + F_\ell \cos(kr - \ell\pi/2)\right] P_\ell(\cos\theta).$$

Equivalently, this may be written

$$\sum_{\ell=0}^{\infty} \frac{G_\ell \sin(kr - \ell\pi/2 + \delta_\ell(k))}{kr} P_\ell(\cos\theta)$$

$$= \frac{1}{2ikr} \sum_{\ell=0}^{\infty} G_\ell \left[e^{i(kr-\ell\pi/2+\delta_\ell)} - e^{-i(kr-\ell\pi/2+\delta_\ell)}\right] P_\ell(\cos\theta).$$

The first term in the above equation is an outgoing spherical wave while the second is an incoming one. But the incoming spherical wave must match the ℓ^{th} incoming part of e^{ikz}. Thus we must have

$$-\frac{1}{2ikr} G_\ell e^{-i(kr-\ell\pi/2+\delta_\ell)} = -\frac{1}{2ikr} i^\ell (2\ell+1) e^{-i(kr-\ell\pi/2)}.$$

Therefore, we get

$$G_\ell e^{-i\delta_\ell} = i^\ell (2\ell+1) = e^{i\pi\ell/2}(2\ell+1)$$

and thus

$$G_\ell = e^{i(\pi\ell/2+\delta_\ell)}(2\ell+1) \tag{1}$$

Also, I should have included the explicit form of the differential scattering cross-section at the bottom of p. 22. It is

$$\frac{d\sigma}{d\Omega} = |f|^2 = \frac{1}{k^2} \sum_{\ell=0}^{\infty} \sum_{\ell'=0}^{\infty} (2\ell+1)(2\ell'+1) e^{i(\delta_\ell - \delta_{\ell'})} \sin\delta_\ell \sin\delta_{\ell'} P_\ell(\cos\theta) P_{\ell'}(\cos\theta).$$

This expression cannot, in general, be simplified further (unlike that for the total cross-section).

Finally, there is a small mistake in the second line of p. 23: there should be a factor of $1/k^2$ in front of the double sum.